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STATISTICAL ANALYSIS OF FATIGUE OF COMPOSITE MATERIALS.(U)

JUL 78 P C CHOU, A S WANG F33615-77-C-5039

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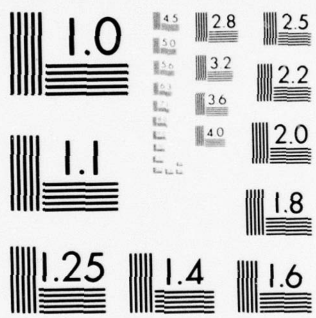

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STATISTICAL ANALYSIS OF FATIGUE OF COMPOSITE MATERIALS

DYNA EAST CORPORATION
227 HEMLOCK ROAD
WYNNEWOOD, PENNSYLVANIA 19096

JULY 1978

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Interim Report for Period 1 April 1977 to 31 March 1978

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AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
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FOR THE COMMANDER

J. M. Kelble

J. M. KELBLE, Chief
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The objectives of the program are to study three basic hypotheses in fatigue of composite materials. These are: 1) cyclic loading degrades the matrix properties and thus lower the compression strength more than tension-strength; 2. the static strength and fatigue life have equal ranks; and 3) the residual strength may increase. Both analytical and experimental approaches are used. The results are not yet conclusive. A sudden-death model is proposed to measure the degradation of residual strength. Three regimes of residual strength are distinguished, these are the increase of strength, weak degradation, and strong degradation.

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Foreword

This is an interim report of a program sponsored by Air Force Materials Laboratory, Air Force System Command, Wright-Patterson Air Force Base, Ohio 45433, under contract F33615-77-C-5039 with Dyna East Corporation. The Air Force Project monitor is Dr. Stephen W. Tsai (FY 1457). Dr. Pei Chi Chou is the principal investigator. He is assisted by Mr. Robert Croman in the theoretical analysis. The experimental phase of the research is carried out by Dr. A.S.D. Wang, with the assistance of Mr. James Alper.

The project is for a duration of 30 months. This report covers work performed during the period April 1, 1977 to March 31, 1978.

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SUMMARY

The objectives of the program are to study three basic hypotheses in fatigue of composite materials. These are: 1. cyclic loading degradates the matrix properties and thus lower the compression strength more than tension-strength, 2. the static strength and fatigue life have equal ranks, and 3. the residual strength may increase. Both analytical and experimental approaches are used. The results are not yet conclusive. A sudden-death model is proposed to measure the degradation of residual strength. Three regimes of residual strength are distinguished, these are the increase of strength, weak degradation, and strong degradation.

Statistical Analysis of Fatigue of Composite Materials

I. Introduction

The goal of this project is to gain understanding of fatigue life and residual strength of unidirectional graphite/epoxy composite. We have limited our study to tension fatigue at constant maximum stress.

In the understanding of composite fatigue, there are a few hypotheses that seem controversial among the researchers in this field. Three of these hypotheses are as follows:

A. Cyclic loading degradates the matrix; reducing its stiffness and strength, causing debonding. Since compression strength is more sensitive to matrix properties than tension strength, therefore fatigue degradates compression strength more than tension strength.

B. The static strength and fatigue life have equal ranks.

C. The residual strength may increase, because the debonding and matrix degradation near crack tips due to fatigue may have a "softening" effect, or creating a larger effective crack tip radius, thus increasing its static strength.

The objectives in the program are to verify or disprove these hypotheses. Our research involves both theoretical study and experimental testing. On the theoretical phase, we have obtained some results and reported them in two publications, which are reproduced here in the Appendices. In Section II below, we summarize the major conclusions in these two publications, and some other preliminary results.

The experimental work has not been completed. In Section III, our general experimental approach and some of the preliminary results are given.

Our plan of research for the next year is presented in Section IV.

II. Theoretical Approach

In this section, we shall give a brief summary of the preliminary results of the theoretical study. These include the ranking methods, parameter estimation, properties depending on percent zero-degree ply and residual strength models.

A. Ranking - In some of the methods of parameter estimation, the experimental data must first be assigned an approximate cumulative distribution, which is known as ranking. In using probability papers, such as normal probability paper and Weibull paper, the ranking of each data point must be estimated first. There are many ranking formulas. If N is the total number of specimens, and n_i is i^{th} order specimen (the 1st order has the lowest value, 2nd order the next lowest, etc.), F_i is the estimated cumulative distribution of the i^{th} order specimen, then we have the following ranking formulas:

simple rank: $F_i = n_i/N$

mean rank: $F_i = n_i/(N+1)$

median rank: $F_i \cong (n_i - 0.3)/(N + 0.4)$

We have made comparison on fatigue life distributions obtained by these ranking methods. The estimated parameters can be quite different by these methods. For instance, a set of fatigue data produced a Weibull shape parameter of 0.61 by simple rank, 0.63 by mean rank, and 0.72 by median rank.

It is felt that the median rank is most suitable and should be used in general.

B. Parameter Estimation - We have concentrated on using two-parameter weibull distribution. Various methods of estimating Weibull parameters from static strength and fatigue life data are studied. Special attention is given to methods that can treat censored data (or suspended item) in fatigue tests.

Methods used include:

1. Cumulative Hazard
2. Hazard Rate
3. Modified Rank-increment and Least Square
4. Maximum Likelihood with Censoring.

The last two methods have been programmed on the computer and can be used routinely. We feel that the maximum likelihood method is the most appropriate.

C. Percent 0°-ply - In studying the shape parameters of existing experimental data of graphite/epoxy composites, we found that the scatter in static strength and fatigue life is larger (small shape parameter) for those laminates that have a large percentage of 0°-ply. The unidirectional, which has 100 0°-ply, has the largest scatter (lowest shape parameter).

D. Residual Strength Models - The degradation model proposed by Hahn and Yang is analyzed, and the corresponding equations of residual strength distribution are derived. A sudden-death model is proposed and compared with the Hahn-Yang model. These results are presented in Appendix A.

It is shown in Appendix A that the Hahn-Yang model is not general enough to describe various types of degradation. We propose a more general degradation equation, which is based on the equal-rank assumption. It has a free parameter that can be adjusted to fit different residual strength data. It is found that the residual strength may increase or decrease. In the latter case, the degradation could be strong or weak. These analyses are given in Appendix B.

III. Experimental Approach

Since the objective of the experiment is to identify the major material characteristics of the unidirectional composite system when subjected to tensile fatigue load, the test program is designed to meet this objective accordingly.

During this reporting period, the experimental work performed may be summarized as follows:

Initially, we have purchased from the Hercules, Inc. 13 panels of AS-3501-06 laminates. Of these panels 10 were 6-ply U.D. laminates, 2 were 8-ply $\pm$ 30° laminates and 1 8-ply $\pm$ 45° laminates. All panels were 1x2 ft^2 in size (30.5x61 cm^2).

Test coupons were cut from these panels yielding 28 tensile coupons (1.9 cm x 22.5 cm) and 28 compression coupons (1.9 cm x 3.2 cm). All tensile coupons were also furnished with glass/epoxy end-tabs (of size 1.9 cm x 3.8 cm). To date, a total of 160 tensile coupons and 80 compression coupons were prepared. Of these, only 92 tensile coupons have so far been fatigue tested.

All fatigue tests were performed using an INSTRON closed-loop hydraulic test machine with programmable load-control. The conditions under which fatigue tests were conducted are as follows:

temperature:	room ambient	23°C
humidity:	room ambient	65%R.H.
load form:	sinusoidal @ 12 Hz (tensile)	

The following table illustrates the types of tests which have been completed during this reporting period:

Nature of Test	Sample population	Status of Test	Remarks
Static Tension to failure	25	completed	Determine static base-line information
Fatigue to failure at $.6S_m$	2 (run out after $2x10^6$ cycles	completed	Determine fatigue limit
Fatigue to failure at $0.8S_m$	20 (2 run of after 10^6)	completed	Determine fatigue failure distribution at $.8S_m$
Fatigue to 10^4 cycle at $.7S_m$	15 (2 failure before 10^4)	completed	Determine residual tensile strength distribution
Fatigue to 10^5 at $0.7S_m$	15 (3 failure before 10^5)	completed	"
Fatigue to 10^6 at $0.7S_m$	15 (5 failure before 10^6)	completed	"

Total tests: 92

IV. Plan for Next Year

During the next reporting period, April 1, 1978 to March 31, 1979, we plan to perform research on the following:

A. Theoretical Approach - The residual strengths formula proposed in Appendix B can accommodate only degradation of residual strength. As mentioned there, quite a few researchers have found increase in residual strength after a moderate number of cycle of fatigue loading. We shall search for a residual strength equation that can accommodate increase in residual strength, as well as strong and weak degradations.

We shall also study the increase of static strength due to local "softening" from a deterministic mechanics point of view.

The maximum likelihood method used now can only accommodate a single Weibull distribution. We shall extend this method to handle distributions that are best fitted by more than one Weibull function. In other words, on the Weibull paper, the distribution is fitted by more than one straight line segment.

The role of the strength-life equal rank assumption will be studied in more detail. Hahn and Kim have demonstrated the general validity of this assumption by applying "proof testing" to glass/epoxy composites subjected to static fatigue. Awerbuch and Hahn have tried the same technique for graphite/epoxy in tension fatigue, with positive, but not conclusive results.
This equal-rank assumption, which is most essential in the current methods of predicting fatigue life and residual strength, has been subject to criticism. We shall try to find means to determine the validity of this assumption.

B. Experimental Approach - Future work in the experimental program has been planned as follows:

1. Proof test. (U.D.)

   a) one excursion to S_m and then to static tension failure (30 specimens minimum):

   b) one excursion to S_m and then fatigue operated at $0.7S_m$ and a certain life (50 specimens minimum).

2. Compression test (U.D.)

   a) Static compression tests. (30 specimens minimum)

   b) tensile fatigue at 0.7Sm to a certain life and then residual tested under compression (50 specimens minimum).

3. Tensile test (static and fatigue on $\pm$ 30° and $\pm$ 45° laminates). Minimum 50 specimens each.

The above listed work is scheduled to be performed during the next 12 month period.

Appendix A. Degradation and Sudden-Death Models of Fatigue of Graphite/Epoxy Composites.

This is the manuscript of a paper presented at the 5th ASTM Conference on Composite Materials: Testing and Design, March 22, 1978, New Orleans, La.

Degradation and Sudden-Death Models of Fatigue of Graphite/Epoxy Composites*

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ABSTRACT: A detailed approach to the degradation and sudden-death models of residual strength is presented. The models were used to predict the residual strength of six sets of experimental data of graphite/epoxy composites. The adequacy of these models was investigated with the use of hypothesis testing and through the study of the weakest residual strength specimens. Both models did a good job in predicting mean residual strength but were overly resistive in predicting the strength of the weakest specimens. The decrease in residual strength was observed to be less for unidirectional composites than for composites of general layup.

KEY WORDS: Graphite/epoxy composite, residual strength, degradation, sudden-death, statistical analysis.

* This work is supported by the Air Force Materials Laboratory, Wright-Patterson AFB, Ohio

I. Introduction

Two models for the residual strength in fatigue tested specimens are studied here, the degradation model and the sudden-death model. The degradation model is based on the assumption used previously in [1] and [2]. It stipulates that the strength of each specimen decreases a little after each cycle of fatigue loading. When the residual strength drops to the value of the applied fatigue stress, fatigue failure occurs. On the other hand, the sudden-death model assumes that the strength of a specimen does not change after each cycle of fatigue loading. The effect of each cycle is impressed on the specimen in a form other than reducing its residual strength. For instance, it may change the matrix properties, which does not change the residual strength immediately. The fatigue failure is governed by some mechanism other than the residual strength. Only when the applied cycles are close to the fatigue life, will the strength then drop drastically in a short number of cycles. For the sudden-death model, we have to impose the additional assumption that there is a unique relation between static strength and fatigue; the stronger ones last longer [2]. This unique relation is implied in the degradation model.

As shown later, some experimental data do agree better with the sudden-death model. Regardless of the direct applicability, the sudden-death model is useful as a limiting case in residual strength study.

In comparing the residual strength with the static strength, it is proposed that the reduced population that includes only "top-percentage" of the static specimens should be used. The percentage that is excluded should equal the percentage of the fatigue specimens that failed before the residual strength

test is taken. By comparing the "top-percent mean" of the static strength with the mean residual strength, we can see whether there is degradation or increase in strength. The top-percent mean can be calculated either from the distribution of the total population by taking proper conditional probability, or by taking the sample mean of the appropriate stronger samples.

In studying the residual strength of composite materials under fatigue loading, Halpin et al [3] proposed a degradation equation that is based on the crack propagation of homogeneous materials. Realizing that fatigue failure of composites is not dictated by the initiation and growth of a dominant crack, Hahn and Kim [1] introduced the concept of rate of change of residual strength. Without referring to any crack, they assumed the time rate of decrease of residual strength is inversely proportional to the residual strength to a certain power. From this deterministic residual strength equation, and the static strength distribution, they derived the fatigue life distribution. Following the same approach as Hahn and Kim, Yang and Liu [2] further derived the residual strength distribution, and compared the results with one group of experimental data.

In the degradation model presented in this paper, we start with the same assumption on rate of degradation as used in [1] and [2]. An approach different from that of [2], however, is used in deriving the fatigue life and residual strength distributions. In deriving the fatigue life distribution, we use as our population those specimens that have a static strength larger than the fatigue stress. By using this population base, the life distribution does not violate the basic properties of a cumulative distribution function.

There is no negative position parameter in the life distribution, and at zero life, the cumulative distribution is zero. In deriving the residual strength distribution, we again limit the population to those specimens that have survived the fatigue test. This is done by taking proper conditional probability. The resulting residual strength distribution behaves nicely; no discontinuous value has to be assigned arbitrarily.

Experimental data on fatigue residual strength of graphite/epoxy composites from four sources were used for comparison with the theoretical models. They contain six sets of residual strength data. We limited our study to tension-tension fatigue at constant amplitude.

II. The Degradation Model of Fatigue

We shall limit our discussion to constant amplitude fatigue, at the constant maximum stress S. We shall assume that the static strength distribution is of the form of a two-parameter Weibull with the parameters known. The fatigue life is assumed measured, with sample lives known, but the exact form of the life distribution is not selected before hand. The theoretical model will yield the form of the life distribution. For a fixed specimen, the degradation model assumes the rate of residual strength degradates according to the strength to a certain power, or

$$\frac{d\,R(n)}{dn} = -\frac{f(S)}{c\,R^{c-1}} \qquad (1)$$

where R(n) is the residual strength after n cycles of fatigue at stress S, c is a positive constant to be determined later, f(S) is a function of S, with positive value. Since we shall consider only one fixed value of S, f(S) is a constant. Integration of Eq. (1) yields

$$R(n) = [R^c(0) - f(S)n]^{1/c} \qquad (2)$$

where R(0) is the static strength of this specimen. The degradation model further stipulates that the fatigue failure occurs when the residual strength is decreased to the value S. If N is the cycle when fatigue failure of this specimen occurs, then we have

$$R(N) = S \qquad (3)$$

The degradation model is represented by Eqs. (1) and (3). Both of these equations are deterministic; they apply to each individual specimen, regardless of the randomness, or the distributions of the strength and life of the population.

It may be pointed out that Eqs. (2) and (3) also imply the one-to-one static fatigue relation discussed in [1] and [2]. This can be seen by substituting N for n in Eq.(2), and utilizing Eq. (3),

$$N = [R^c(0) - S^c]/f(S) \tag{4}$$

Since S and f(S) are constants, the specimen that has larger value of static strength R(0) will also have a longer life N.

We shall next derive the life and residual strength distributions from the degradation equations (1) and (3), and the static strength distribution. Let the static strength be a two-parameter Weibull, with the cumulative distribution function

$$F_{R(0)}(x) = P[R(0) \leq x]$$

$$= 1 - \exp[-\left(\frac{x}{\beta}\right)^{\alpha}], \quad 0 \leq x < \infty \tag{5}$$

where α is the shape parameter, and β the scale parameter, or characteristic strength. We use R(0) to represent the random variable of static strength and x to represent its value. The lower range of the strength x is at zero. This implies that some specimens, even though only a very small percentage, will fail statically below the stress S. The fatigue life is measured at the stress S. Therefore, the population represented by the life distribution is not the complete population of $0 \leq x < \infty$; it is the population with strength larger than S. In order to obtain the life distribution from the strength distribution, we formulate the distribution of strength for those with strength larger than S, by taking a conditional probability, or

$$F_{R(0),S}(x) = P[R(0) \leq x | R(0) > S]$$

$$= \frac{P[S < R(0) \leq x]}{P[R(0) > S]}$$

$$= 1 - \exp[-\left(\frac{x}{\beta}\right)^{\alpha} + \left(\frac{S}{\beta}\right)^{\alpha}] \tag{6}$$

From here on, whenever static strength distribution is mentioned, it refers to Eq. (6).

Let $F_N(n)$ be the cumulative distribution of fatigue life at stress level S, with N as the random variable of life, and n its value, then

$$F_N(n) = P[N \leq n] \tag{7}$$

Substituting Eq. (4) into (7), we get

$$F_N(n) = P[R(0) \leq \{nf(S) + S^c\}^{1/c}] \tag{8}$$

According to Eq. (6), this becomes

$$F_N(n) = 1 - \exp\{-\left[\frac{n}{n_0} + \left(\frac{S}{\beta}\right)^c\right]^{\alpha/c} + \left(\frac{S}{\beta}\right)^{\alpha}\} \tag{9}$$

where

$$n_0 = \beta^c/f(S) \tag{10}$$

may be considered as a characteristic life. This is the life distribution based on the strength distribution Eq. (6) and the degradation equation (4). Values of the constants α and β are known from the strength distribution; values of the two constants n_0 and c must be estimated from fatigue life experimental sample data.

Note that the domain for life n is from 0 to ∞, and that $F_N(0) = 0$, and $F_N(\infty) = 1$, consistent with the basic properties of a distribution function. Equation (9) may seem to be a three-parameter Weibull distribution, but it is not. Note it has no position parameter, or $F_N(n) = 0$ at $n = 0$. The term S/β is a known constant, not a parameter to be determined. It has only two parameters, n_0 and c, and can be called a modified two-parameter Weibull distribution.

With a change of notation, it can be seen that Eq. (9) is identical to Eq. (10) of Hahn and Kim [1]; it is different from Eq. (8) of Yang and Liu [2], which is a three-parameter Weibull distribution with a negative position parameter.

The value of c is in the neighborhood of 10. For values of $(S/\beta) < 0.7$, values of n_0 and α/c could be determined approximately by fitting experimental data to an exact two-parameter Weibull; in other words, (S/β) may be neglected in estimating the parameters n_0 and α/c. For large values of S/β, the test data may have to be fitted to the distribution of the exact form of Eq. (9).

The residual strength at a given life is, by definition, for those specimens that survived this fatigue life. The population for the residual strength distribution includes only the survivors. Let n_γ be the life, or cycle, at which the residual strength is measured, and at this life, $(1-\gamma)$x100 percent of the specimens failed, γx100 percent survived. From Eq. (4) and the definition of n_0, we see that the specimen with a life of $N = n_\gamma$, must have a static strength of $R(0) = x_\gamma$, such that

$$\frac{n_\gamma}{n_0} = \left(\frac{x_\gamma}{\beta}\right)^c - \left(\frac{S}{\beta}\right)^c \tag{11}$$

It follows from Eqs. (6) and (9) that

$$1-\gamma = F_N(n_\gamma) = 1 - \exp\left\{-\left[\frac{n_\gamma}{n_0} + \left(\frac{S}{\beta}\right)^c\right]^{\alpha/c} + \left(\frac{S}{\beta}\right)^\alpha\right\}$$

$$= 1 - \exp\left\{-\left(\frac{x_\gamma}{\beta}\right)^\alpha + \left(\frac{S}{\beta}\right)^\alpha\right\} = F_{R(0),S}\,(x_\gamma) \qquad (12)$$

Therefore, at the fatigue cycle n_γ the survivors are those specimens that are the top γx100 percent in static strength among those with strength above S. Using the notation $R(n_\gamma)$ for the random variable of the residual strength after n_γ cycles, and y for its value, we have,

$$F_{R(n_\gamma)}(y) = P[R(n_\gamma) \leq y | N > n_\gamma]$$

$$= P[R(n_\gamma) \leq y | R(0) > x_\gamma] \qquad (13)$$

Substituting Eq. (2) for $R(n_\gamma)$, we get

$$F_{R(n_\gamma)}(y) = P\left[R(0) \leq \left\{y^c + f(S)n_\gamma\right\}^{1/c} \Big| R(0) > x_\gamma\right] \qquad (14)$$

Application of Eqs. (6), (10) and (11) yields

$$F_{R(n_\gamma)}(y) = 1 - \exp\left\{-\left[\left(\frac{y}{\beta}\right)^c + \left(\frac{x_\gamma}{\beta}\right)^c - \left(\frac{S}{\beta}\right)^c\right]^{\alpha/c} + \left(\frac{x_\gamma}{\beta}\right)^\alpha\right\}$$

$$= 1 - \exp\left\{-\left[\left(\frac{y}{\beta}\right)^c + \frac{n_\gamma}{n_0}\right]^{\alpha/c} + \left(\frac{x_\gamma}{\beta}\right)^\alpha\right\} \qquad (15)$$

This is the residual strength distribution. Once the static strength and fatigue life are known and the degradation model assumed, all constants in Eq. (15) are known. A comparison of this equation with the experimentally measured residual strength will serve as a verification, or test, of the degradation model.

Note that Eq. (15) is also a modified two-parameter Weibull distribution. The domain is $S \leq y < \infty$, with $F_{R(n_\gamma)}(S) = 0$, and $F_{R(\gamma_\gamma)}(\infty) = 1$.

The residual strength distribution, Eq. (15), may also be derived from the static strength distribution by a transformation of variables. Let us first derive from either Eq. (5) or Eq. (6), the distribution of the top γ-Percent of the static strength as

$$F_{R(0),\gamma}(x) = P[R(0) \leq x | R(0) > x_\gamma]$$

$$= 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha + \left(\frac{x_\gamma}{\beta}\right)^\alpha\right] \tag{16}$$

The corresponding density function is

$$f_{R(0),\gamma}(x) = \frac{dF}{dx} = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha + \left(\frac{x_\gamma}{\beta}\right)^\alpha\right] \tag{17}$$

Next, we shall write the degradation equation (2) in terms of y and x, or

$$y = [x^c - f(S)n]^{1/c} \tag{18}$$

By taking the derivative dx/dy from Eq. (18), and the transformation equation

$$f_{R(n_\gamma)}(y) = f_{R(0),\gamma}(x)\,\frac{dx}{dy} \tag{19}$$

we obtain,

$$f_{R(n_\gamma)}(y) = \frac{\alpha}{\beta}\left(\frac{y}{\beta}\right)^{c-1} \left[\left(\frac{y}{\beta}\right)^c + \left(\frac{x_\gamma}{\beta}\right)^c - \left(\frac{S}{\beta}\right)^c\right]^{(\alpha-c)/c}$$

$$\cdot \exp\left\{-\left[\left(\frac{y}{\beta}\right)^c + \left(\frac{x_\gamma}{\beta}\right)^c - \left(\frac{S}{\beta}\right)^c\right]^{\alpha/c} + \left(\frac{x_\gamma}{\beta}\right)^\alpha\right\} \tag{20}$$

Upon integration of Eq. (20), Eq. (15) results. The mean of the residual strength, μ_γ, may be obtained from Eq. (20) by numerically integrating the equation

$$\mu_\gamma = \int_S^\infty y \, f_{R(n_\gamma)}(y)\,dy \tag{21}$$

III. The Sudden-Death Model

In the sudden-death model, the residual strength of each specimen is assumed to remain unchanged, or

$$R(n) = R(0) \tag{22}$$

This equation replaces Eqs. (2) and (3) of the degradation model. The static-fatigue one-to-one relation will be assumed to hold. Recall that in the degradation model the one-to-one relationship is implied by Eqs. (2) and (3). In the present case, we abandon Eqs. (2) and (3), but still want to retain the one-to-one relationship. The static strength distribution will be the same as in Eq. (6). The fatigue life distribution may be of any form, and would be most logical to take a two-parameter Weibull,

$$F_N(n) = 1 - \exp\left[- \left(\frac{n}{n_0}\right)^{\alpha/c} \right] \tag{23}$$

For ease of comparison, however, we shall use a life distribution of the form of Eq. (9). The one-to-one relationship is then characterized by

$$F_{R(0),S}(x_\gamma) = F_N(n_\gamma) \tag{24}$$

Note that Eq. (12), which is identical to (24), is a derived relation; whereas here Eq. (24) is a basic assumption. It follows that after n_γ cycles of fatigue 100γ percent of the specimen survive, and contribute to the residual strength. The residual strength distribution can be derived by writing Eq. (22) as

$$y = x \tag{25}$$

substituting Eqs. (17) and (25) into Eq. (19) and integrating, we get

$$F_{R(n_\gamma)}(y) = F_{R(0),\gamma}(x) \tag{26}$$

The mean of the residual strength based on the sudden-death model can then be integrated from

$$\mu_\gamma = \int_{x_\gamma}^{\infty} x \, f_{R(0),\gamma}(x) \, dx \tag{27}$$

IV. Experimental Data

In order to study the applicability of the degradation and sudden death theories in predicting residual strength, groups of data were obtained from four different sources. All the data used was for graphite/epoxy composite specimens subjected to tension loading. These tests may be categorized by the percent of 0° plies comprising the specimens. The first category of tests were obtained from work by Yang and Liu [2] and Ryder and Walker [4] (Laminate I) and the percentage of 0° plies is 25%. The second category, which has 67% 0° plies, is the Laminate II data of Ryder and Walker. The third category of data comes from the work of Awerbuch and Hahn [5] and Wang [6]. Here the specimens are unidirectional (100% 0° ply).

All these sets of data are tabulated in Appendix A. For each set the exact layup is given. The static strength, fatigue life and residual strength data are presented in that order. For the fatigue data the maximum applied load, S, is given along with the stress ratio, R, and the cycling frequency, F. As is noted in the tables, fatigue life in parenthesis denotes an element that was suspended or censored at that particular life for the purpose of residual strength test or was a run-out. The data of the residual strength tests which were used for the purposes of this report are listed along with the conditions and life to which they were fatigued.

In order to predict the residual strength by the theories presented in this report it was necessary to fit Eqs. (5) and (9) to the static and fatigue data respectively. Eq. (5) is a two parameter Weibull and presents no difficulties in order to determine α and β. Eq. (9) is a modified two parameter Weibull distribution and is hard to fit to the fatigue data. However the S/β ratios considered in this paper are all less than or equal to 0.7. Since the exponents c and α are of the order of 10, the S/β terms become small and can

be neglected. With this simplification the determination of the two unknown parameters, c and n_0 is straight forward. The maximum likelihood method of Cohen [7] was used to this end. This technique can handle progressively censored specimens. Before presenting the obtained parameters, a few comments must be made on the treatment of some of the data.

The treatment of the static data was straight forward but several sets of fatigue data were open to interpretation and will be discussed here. The fatigue life parameters for the Awerbuch-Hahn data were determined by assuming that the last 16 censored items were suspended at a life of 600,000 cycles instead of 2 million cycles. The reason for this is that in private communications with those authors there was reason to believe that there were more failures past the last failed specimen. Therefore in order to be fair with the data at hand it was decided to censor those mentioned items at an earlier life. The fatigue parameters obtained for the Laminate I of Ryder-Walker were obtained by combining data sets i, ii, and iii. In the treatment of Laminate II data of Ryder-Walker it was decided to ignore the three failures at 1 cycle. Inclusion of these three data points would have resulted in an unrealistically large value of n_0. Data sets i and ii of the Yang-Liu fatigued specimens were combined for analysis. The estimated parameters for the static and fatigue samples are summarized in Table 1.

V. Comparison between Experimental Data and Theoretical Models

Having estimated the static strength and fatigue life Weibull parameters, we are now ready to see how well the theoretical models predict the residual strength. We shall first compare the means of the residual strength, and then compare the strength of the weakest specimens.

The mean of the residual strength for six cases has been calculated by the degradation model with Eq. (21), and by the sudden death model with Eq. (27). These are tabulated in Table 2. The means have been normalized by the respective static strength scale parameters, β. The sample mean, $\bar{x}$, and sample standard deviation, s, are also recorded for each case. As mentioned before, the sudden death model represents no degradation of the individual specimens. Comparing the values of the mean of the degradation model with those of the sudden death model, we see that the decrease of the mean is very small. For all six cases studied here, the mean of the degradation model is within three percent of those of the sudden death model. Comparing the experimental sample mean with the sudden death means, we see that two of the cases, both of unidirectional layup (100% 0° ply), have a slight increase in residual strength, although the increase is so small that it may be within the statistical scatter. For these two cases, the sample mean is closer to the sudden death means; for the other four cases, the sample mean is closer to the mean of the degradation model. Since the number of specimens used in these cases are not the same, hypothesis testing will be made to indicate which model is in agreement with the experimental data.

The hypothesis to be tested is that the population from which the sample was obtained has a mean μ which is the same as the theoretical mean μ_{γ}.

This hypothesis is denoted as

$$H_0: (\mu = \mu_\gamma) \tag{28}$$

The alternative hypothesis is denoted

$$H_A: (\mu < \mu_\gamma) \text{ or } H_A: (\mu > \mu_\gamma) \tag{29}$$

The choice of the alternative will be determined by the relative values of $\bar{x}$ and μ_γ. The random variable

$$\frac{(\bar{x} - \mu_\gamma)\sqrt{n}}{s} \tag{30}$$

where n is the sample size, is assumed to have a Student-t distribution with n-1 degrees of freedom. Strickly speaking, this is true only when the population is normal. For practical purposes, it is a good enough approximation for non-normal distributions, [8]. Now the hypothesis H_0 will be accepted at a significance level θ if

$$\left| \frac{(\bar{x} - \mu_\gamma)\sqrt{n}}{s} \right| < t_{\theta,n-1} \tag{31}$$

where $t_{\theta,n-1}$ is the θ-percentile value of the t-distribution with (n-1) degree of freedom. Otherwise the alternative hypothesis, H_A is accepted and H_0 rejected.

For a significance level of 5% the degradation model is acceptable for all six cases, as shown in Table 3. The sudden death model is also acceptable to all except one case. When the significance level is increased to 10%, the degradation model is still acceptable for five cases, and is not acceptable for one of the unidirectional cases. With 10% significance level, the sudden death model is acceptable for two of the three unidirectional cases and for only one of the other three cases. It must be kept in mind that by increasing

the significane level from 5% to 10%, we decrease the "type II" error but increase the "type I" error. The simple hypothesis test used here can give only a general indication.

From the discussion above we may draw the tentative conclusion that the degradation model is satisfactory in predicting the mean residual strength. For most cases, the sudden death model may also be used to predict the mean residual strength. For two of the unidirectional composites, the sudden death model agrees with the experimental data better than the degradation model.

We shall now make the comparison on the values of the weakest residual strength specimen. The present degradation model has a degradation rate that is highly dependent on the strength itself. As can be seen from Eq. (1), at high values of $R(n)$, dR/dn is small; the only appreciable decrease in strength occurs when $R(n)$ is small, either by having a small value of $R(0)$ to start with, or when n is large. Figure 1 contains plots of Eq. (2) with Wang's static strength and fatigue data. As can be seen, at 10^5 cycles, the stronger specimens have very little decrease in strength according to the degradation model. At this cycle, most of degradation occurs for the weaker specimens, those having a static strength between 0.8 to 0.95. Figure 2 shcws the same trend for the Ryder-Walker Laminate II data. To compare the residual strength models, it is more instructive to compare the weakest of the residual specimens. It may also be mentioned that the weaker residual strength specimens are of the most practical importance.

We shall compare the weakest specimen by first ploting the experimental residual strength data points directly, as solid dots, as shown in Fig. 3.

In this case, there are 18 residual strength data points, which are all shown in the figure. The static strength data are also shown as solid dots, and the fatigue failure points are shown as circles. Next, we obtain the median rank values for 18 specimens from median rank tables, [9]. These median ranks are considered as the distribution function F, and the residual strength corresponding to each of these F values is then calculated from Eq. (15) for the degradation model, and from Eq. (16) for the sudden-death model. In Fig. 3, the degradation points are plotted as triangles, and the sudden-death points as crosses.

From Fig. 3 it can be seen that the experimental weakest specimen (lowest dot) is in between the weakest sudden-death points and weakest degradation points. For this case, the experiment seems to agree better with the sudden-death model

Figure 4 shows a similar plot for the case of Laminate II of Ryder-Walker. Here, it is interesting to note that the weakest of the 15 specimens from the degradation model is about the same value as that from the sudden death model. The experimental one has a much lower value than the models. This indicates that the present degradation model does not have enough degradation; it is more close to the sudden-death model whereas the experimental data shows more degradation.

Another case is shown in Fig. 5. Again, the degradation model is not much different from the sudden death model. However, the experimental weakest specimen shows a much higher value than those of the models.

VI. Limitation of the Present Degradation Model

The present degradation model has a degradation rate that contains two parameters, f(s) and c, as shown in Eqs. (1) and (2). These two parameters are determined completely by the fatigue life distribution data. For a given material under a given fatigue loading, once the static strength and the fatigue life distributions are known, the residual strength at all fatigue cycles are fixed according to the present model; there is no open parameter to accommodate different residual strength distributions. In other words, according to the present model, two materials having the same static strength and fatigue life distributions, must also have the same residual strength distribution. This seems to be very restrictive.

Application of the present degradation model to the six cases studied shows clearly this limitation. For instance, in Fig. 6, the mean residual strength according to both sudden-death and degradation models are plotted against fatigue cycles, for the case where c is 56.1. The degradation curve is very close to the sudden-death curve, indicating that the degradation model offered very little degradation in this case. This fact has also been shown in Fig. 4. Figure 7 shows another case where c is 10.8. Here, the degradation curve is substantially different from the sudden-death curve, but the experimental data show better agreement with the sudden-death.

In view of the above, we feel that the present degradation model is overly restrictive and not adequate. An appropriate degradation model should have one more parameter to adjust for different residual strength when the static strength and fatigue life are fixed.

VII Conclusions

We shall summarize a few comments made before:

1. In deriving the statistical distribution for strength and life, care must be taken to use the proper population base. It will avoid getting distributions with negative Weibull position parameters, or getting non-zero cumulative distributions at the lower bound of the domain.

2. In general, the degradation model correctly predicts the mean of the residual strength of the six sets of test data studied. This is not a severe test for the model, because the decrease in the mean residual strength in these six cases is very small. The sudden-death model, which assumes no degradation for individual specimens, is also satisfactory in predicting the residual mean for most of these six cases.

3. The sudden-death model should be used as a standard to measure degradation.

4. The degradation model does not predict accurately the weakest residual strength among a set of residual strength data.

5. The degradation model presented here is overly restrictive. Once the static and life distributions are given, it predicts a fixed residual strength distribution, which may not agree with experimental data. A more general degradation model with an additional parameter would be more versatile in matching different residual strength distributions of composite materials.

6. For unidirectional composites, the decrease of residual strength is less than that for composites of general layup.

VIII. Acknowledgement

The authors wish to thank Dr. S. W. Tsai for his encouragement and helpful discussions. They also wish to express their appreciation to Dr. G. P. Sendeckyj for providing them with a parameter estimation program, and to Mr. J. Alper for performing part of the calculations.

Appendix A   Experimental Data of Composite Materials

A.1 Awerbuch-Hahn [5], GR/E, 8-10 ply, 0°

a. Static Tension Strength, ksi† (48 specimens)

122.3	168.0	188.5	205.0	212.7	222.4
123.6	174.8	193.2	205.3	214.5	223.0
147.1	181.5	196.9	205.4	216.8	225.2
149.5	182.3	197.1	206.5	217.4	226.5
149.7	183.0	200.4	207.9	219.4	227.8
161.9	183.6	201.9	209.0	220.0	228.3
161.9	184.7	202.2	211.8	220.6	228.6
166.5	186.4	203.0	212.2	221.5	232.0

$\bar{x}$ = Sample mean strength = 196.5

s = Sample standard deviation = 27.5 ksi

b. Tension-Tension Fatigue Life, Cycles, Max. Stress = 127.3 ksi

R = 0.1, F = 33 Hz (74 specimens, 19 failures, 55 suspended)

10,900	(100,000)*	219,900	(500,000)
16,000	18 { ⋮	223,500	21 { ⋮
17,100	⋮	305,300	(500,000)
31,800	(100,000)	315,400	546,600
33,100	118,200	355,800	(2,000,000)
46,100	118,500	386,500	⋮
47,100	121,600		16 { ⋮
94,000	213,000		(2,000,000)

c. Residual Tension Strength, ksi, at $1x10^5$ cycles. (18 specimens)

Tension-Tension Fatigued with Max. Stress = 127.3 ksi, R = 0.1, F = 33 Hz.

167.2	176.6	198.2	216.6	220.8	227.7
167.7	185.2	212.4	218.2	223.5	230.5
174.8	192.6	213.4	218.4	223.9	230.6

d. Residual Tension Strength, ksi, at $5x10^5$ cycles. (21 specimens)

Tension-Tension Fatigued with Max. Stress = 127.3 ksi, R = 0.1, F = 33 Hz.

164.0	178.4	188.6	200.8	204.8	215.2	221.0
167.1	180.5	192.4	202.4	207.0	217.4	227.2
172.9	182.4	199.3	203.1	207.2	218.6	240.7

† 1 ksi = $6.894x10^6$ Pa

★ Parenthesis indicate element did not fail in fatigue but was suspended at the stated life for residual strength test or run-out.

A.2 Ryder-Walker Laminate I Data [4] GR/E, 16 ply, $(0/+45/90/-45_2/90/+45/0)_S$

a. Static Tension Strength, ksi (25 specimens)

62.0	66.0	69.3	70.6	72.0
64.4	66.0	69.7	70.6	72.6
64.6	67.6	69.9	71.3	73.0
64.6	68.0	69.9	71.4	74.2
65.2	69.3	69.9	71.8	75.4

$\bar{x}$ = Sample Mean Strength = 69.2 ksi

s = Sample Standard deviation = 3.4 ksi

b. Tension-Tension Fatigue Life, Cycles, Max. Stress = 50 ksi, R = 0, F = 10 Hz.

i. Data Set 1 (20 specimens, 20 failures)

11,491	51,848	64,070	81,571
17,578	54,187	69,711	87,373
40,270	58,530	70,049	116,667
41,200	59,320	70,497	367,644
44,830	60,912	71,400	512,600

ii. Data Set 2 (21 specimens, 1 failure, 20 suspended)

21,600

$$20\begin{cases}(31{,}400)\\ \vdots\\ (31{,}400)\end{cases}$$

iii. Data Set 3 (23 specimens, 3 failures, 20 suspended)

5,350
14,200
28,800

$$20\begin{cases}31{,}400\\ \vdots\\ 31{,}400\end{cases}$$

c. Residual Tension Strength, ksi, at 31,400 cycles. (20 specimens)

Tension-Tension Fatigued with Max. Stress = 50 ksi, R = 0, F = 10 Hz.

54.6	59.0	61.6	66.0	67.5	69.9	71.7
57.7	59.8	64.4	66.2	68.8	71.4	72.6
57.9	60.6	64.4	67.0	69.3	71.7	

A.3 Ryder-Walker Laminate II Data [4], GR/E, 24 ply, $(0/+45/0_2/-45/0_2/+45/0_2/-45/0)_S$

a. Static Tension Strength, ksi (20 specimens)

118.7	136.4	141.2	148.7
129.8	136.6	145.4	148.8
133.4	139.5	146.6	150.2
134.5	140.4	147.2	150.5
136.1	140.8	147.6	161.8

$\bar{x}$ = Sample mean strength = 141.7

s = Sample standard deviation = 9.3

b. Tension-Tension Fatigue Life, Cycles, Max. Stress = 100 ksi

R = 0, F = 10 Hz (51 specimens, 9 failures, 42 suspended)

1	930,000
1	38 { (1,000,000) ⋮ (1,000,000)
1	(1,055,000)
910	(1,212,000)
1,880	(1,358,000)
2,000	(1,470,000)
495,040	
826,940	

c. Residual Tension Strength, ksi, at 1×10^6 cycles. (20 specimens)

Tension-Tension Fatigued with Max. Stress = 100 ksi

R = 0, F = 10 Hz.

120.9	138.5	140.6	149.1	152.8
127.5	139.3	142.2	149.6	153.0
134.4	139.9	144.3	150.1	154.5
136.2	140.4	147.7	150.3	159.2

A.4 Yang-Liu [2], GR/E, 8 ply, $(0,90, \pm 45)_S$

a. Static Tension Strength, ksi (12 specimens)

63.152	72.323	77.743	81.324
66.312	72.626	78.316	81.742
71.900	75.050	80.052	84.154

$\bar{x}$ = Sample mean strength = 75.391 ksi

s = Sample standard deviation = 6.394 ksi

b. Tension-Tension Fatigue Life, Cycles, Max. Stress = 52,716 ksi
R = 0.1, F = 20 H_z

i. Data Set 1 (9 specimens, 9 failures)

3,840	117,580
18,790	155,000
88,000	221,200
	228,500
	228,700
	310,000

ii. Data Set 2 (8 specimens, 1 failure, 7 suspended)

17,540

$$7 \left\{ \begin{matrix} (26,000) \\ \vdots \\ (26,000) \end{matrix} \right.$$

c. Residual Tension Strength, ksi, at 26,000 cycles. (7 specimens)
Tension-Tension Fatigued with Max. Stress = 52,716 ksi, R = 0.1, F = 20 Hz.

62.2	74.3
66.3	78.0
69.1	84.1
73.7	

A.5 Wang [6], GR/E, 6 ply, 0°

a. Tension Static Strength, ksi (24 specimens)

158.9	189.3	206.1	214.1	224.0	237.0
177.1	194.2	207.2	214.8	228.2	245.0
177.9	195.4	209.5	216.3	228.6	250.7
186.7	205.3	213.3	222.5	229.7	255.2

$\bar{x}$ = Sample mean strength = 212.0

s = Sample standard deviation = 24.0

b. Tension-Tension Fatigue Life, Cycles, Max Stress = 148.3 ksi

R = 0.1, F = 9.5 Hz (36 specimens, 4 failures, 32 suspended)

8,352	68,517	441,030
15 { (10,000)	15 { (100,000)	531,170
:	:	(1,000,000)
(10,000)	(100,000)	(1,000,000)

c. Residual Tension Strength, ksi, at 1x10^5 cycles. (15 specimens)

Tension-Tension Fatigued with Max. Stress = 146.3 ksi

R= 0.1, F = 9.5 Hz.

194	206	217	229	232
195	215	221	229	238
202	216	221	229	250

References

1. Hahn, H.T. and Kim, R.Y., "Proof Testing of Composite Materials," Jour. Composite Materials, Vol. 9, No. 3, July, 1975, pp. 297-311.

2. Yang, Jann N. and Liu, M.D., "Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates," AFML-TR-76-225, Wright-Patterson AFB, 1976

3. Halpin, J.C., Jerina, K.L., and Johnson, T.A., "Characterization of Composites for the Purpose of Reliability Prediction," Analysis of Test Methods for High Modulus Fibers and Composites, ASTM STP 521, American Society for Testing and Materials, 1973, p. 5.

4. Ryder, J.T. and Walker, E.K., "Ascertainment of the Effect of Compressive Loading on the Fatigue Lifetime of Graphite Epoxy Laminates for Structural Application," AFML-TR-76-241, Wright-Patterson AFB, 1976.

5. Awerbuch, J. and Hahn, H.T., "Fatigue and Proof-Testing of Unidirectional Graphite/Epoxy Composite," in Fatigue of Filamentary Composite Materials, STP 636, American Soc. for Testing and Materials, 1977, pp. 248-266.

6. Wang, A.S.D., private communication, Drexel University test data.

7. Cohen, A.C., "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and on Censored Samples," Technometrics, Vol. 7, No. 4, Nov. 1965, pp. 579-588.

8. Guttman, I., Wilks, S.S., and Hunter, J.S., <u>Introductory Engineering Statistics</u>, John Wiley and Sons, New York, 1971.

9. Johnson, L.G., "The Median Ranks of Sample Values in their Population with an Application to Certain Fatigue Studies," Industrial Mathematics, Vol. II, 1951, pp. 1-9.

TABLE 1 Weibull Parameters by Maximum Likelihood Estimation

Data Source [Ref.]		Static Strength: Shape Parameter α	Static Strength: Scale Parameter, β ksi (GPa)	Static Strength: Sample Mean $\bar{x}/\beta$	Static Strength: Weibull Mean μ/β	Fatigue Life: Shape Parameter α/c	Fatigue Life: Scale Parameter n_0 cycles	Fatigue Life: Ratio of Shape Parameters, c
Yang-Liu	[2]	15.3	78.1 (0.54)	0.965	0.966	1.18	1.65×10^5	13.0
Ryder-Walker Laminate I	[4]	23.9	70.7 (0.49)	0.979	0.978	1.31	1.28×10^5	18.2
Ryder-Walker Laminate II	[4]	16.8	145.9 (1.01)	0.971	0.969	0.30	8.54×10^8	56.1
Awerbuch-Hahn	[5]	9.7	207.5 (1.43)	0.947	0.950	0.75	1.92×10^6	12.9
Wang	[6]	10.2	222.3 (1.53)	0.954	0.952	0.94	1.27×10^6	10.8

TABLE 2 Comparison of Mean Residual Strength

% 0° ply	Data Source [Ref.]	n_γ cycles	Predicted Mean Strength, μ_γ/β Degradation Model	Predicted Mean Strength, μ_γ/β Sudden Death Model	Sample Values Mean, $\overline{x}/\beta$	Sample Values S.D., s/β
25%	Yang-Liu [2]	26,000	0.960	0.985	0.928	0.096
	Ryder Walker Laminate I [4]	31,400	0.967	0.993	0.963	0.081
67%	Ryder Walker Laminate II [4]	$1x10^6$	0.987	0.988	0.983	0.065
100%	Awerbuch Hahn [5]	$1x10^5$	0.964	0.977	0.990	0.108
		$5x10^5$	0.976	1.012	0.962	0.098
	Wang [6]	$1x10^5$	0.960	0.977	0.990	0.071

TABLE 3 Test of the Hypothesis H_0: $(\mu = \mu_\gamma)$, for Mean Residual Strength

Does the sample come from a population whose mean is the same as the theoretical model, or, does the test data agree with the model, in terms of mean residual strength.

Significance Level	Model	Yang-Liu	Ryder Walker Laminate I	Ryder Walker Laminate II	Awerbuch Hahn $n_\gamma=1x10^5$ cycles	Awerbuch Hahn $n_\gamma=5x10^5$ cycles	Wang
5%	Degradation	Yes	Yes	Yes	Yes	Yes	Yes
	Sudden-Death	Yes	Yes	Yes	Yes	No	Yes
10%	Degradation	Yes	Yes	Yes	Yes	Yes	No
	Sudden Death	No	No	Yes	Yes	No	Yes

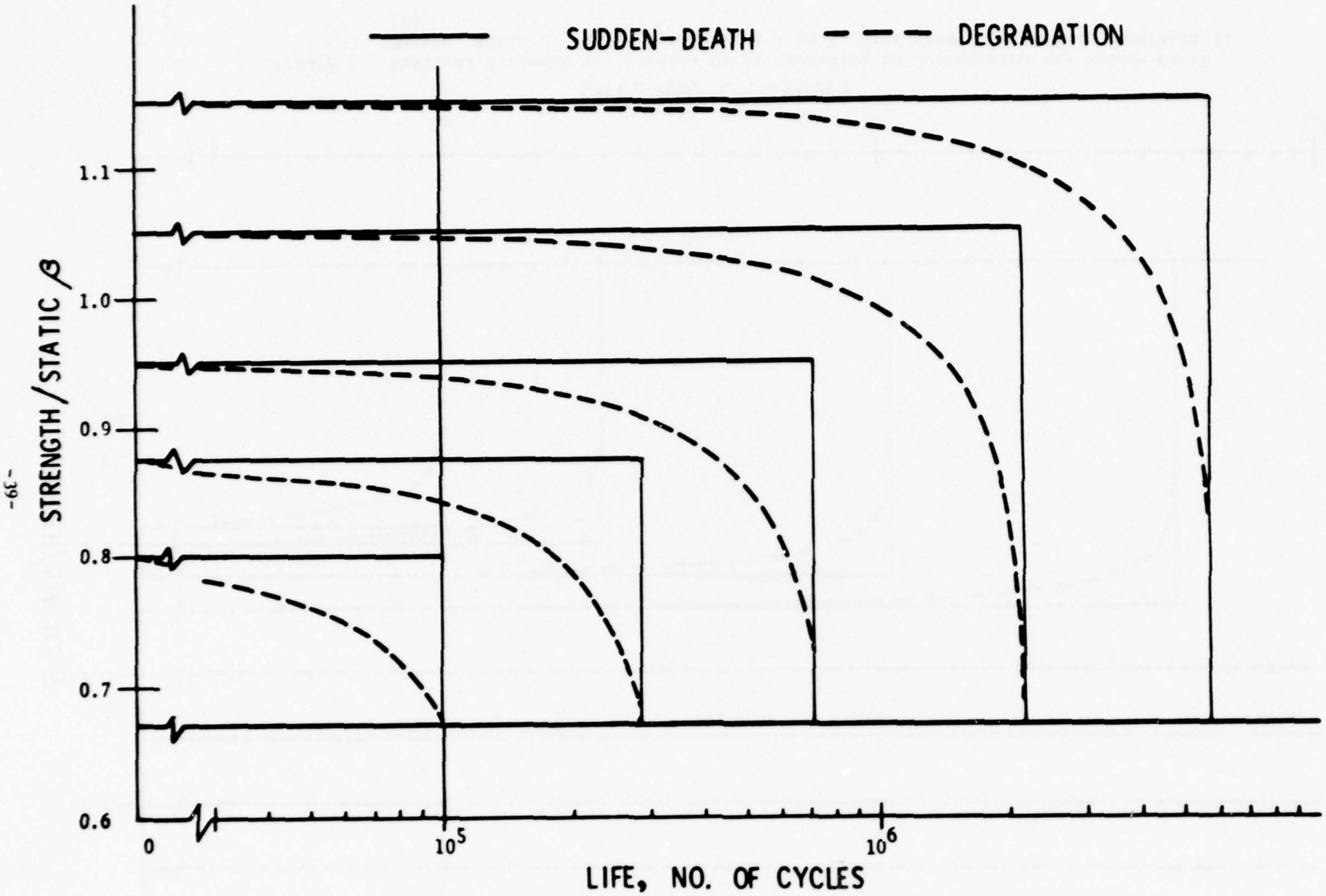


Figure 1. Residual Strength vs. Fatigue Cycle According to Degradation and Sudden-Death Models. Ratio of Shape Parameter, c = 10.8, Data from Wang [6].

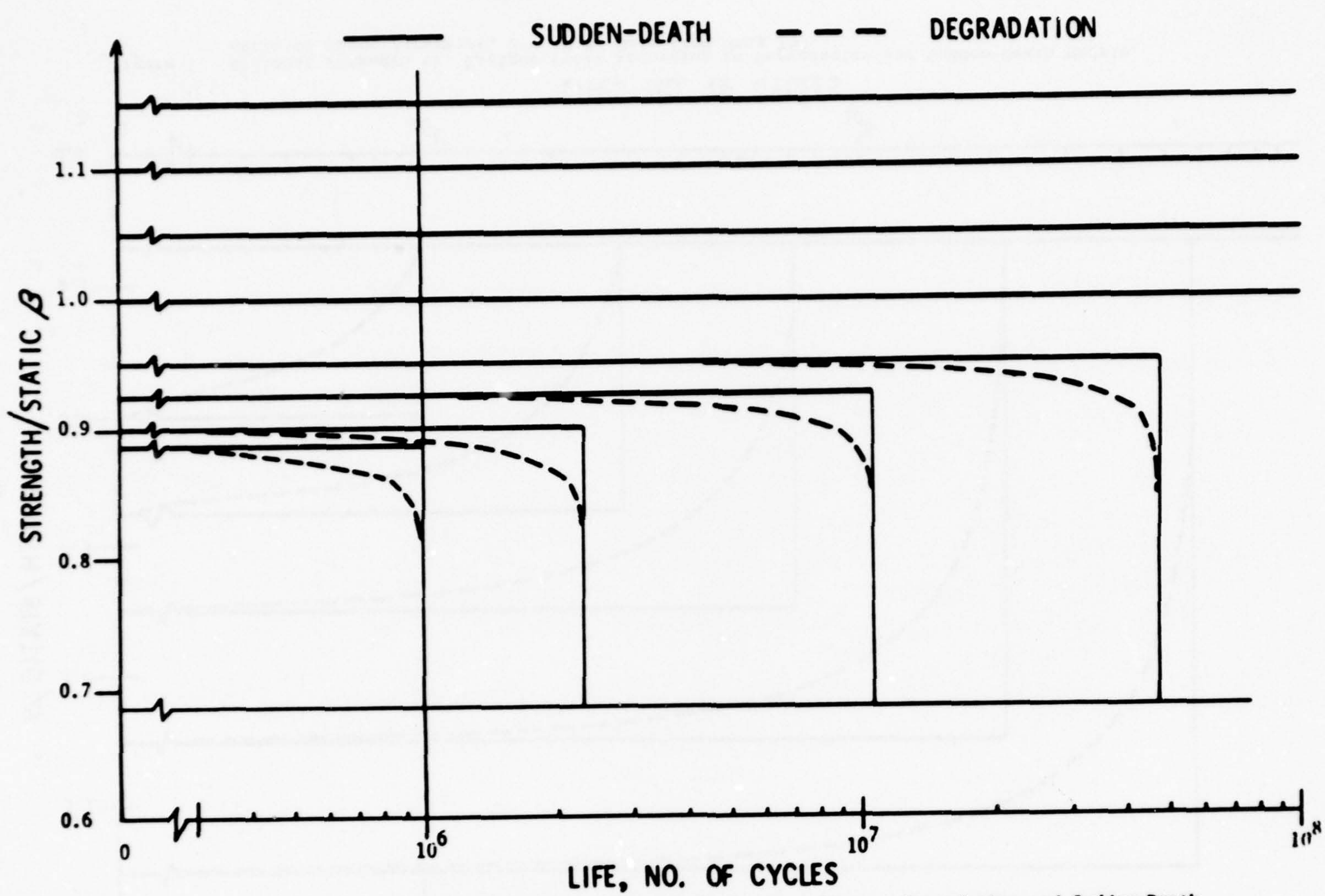


Figure 2. Residual Strength vs. Fatigue Cycle According to Degradation and Sudden-Death Models. Ratio of Shape Parameter, c = 56.1, Data from Ryder-Walker, Laminate II [4].

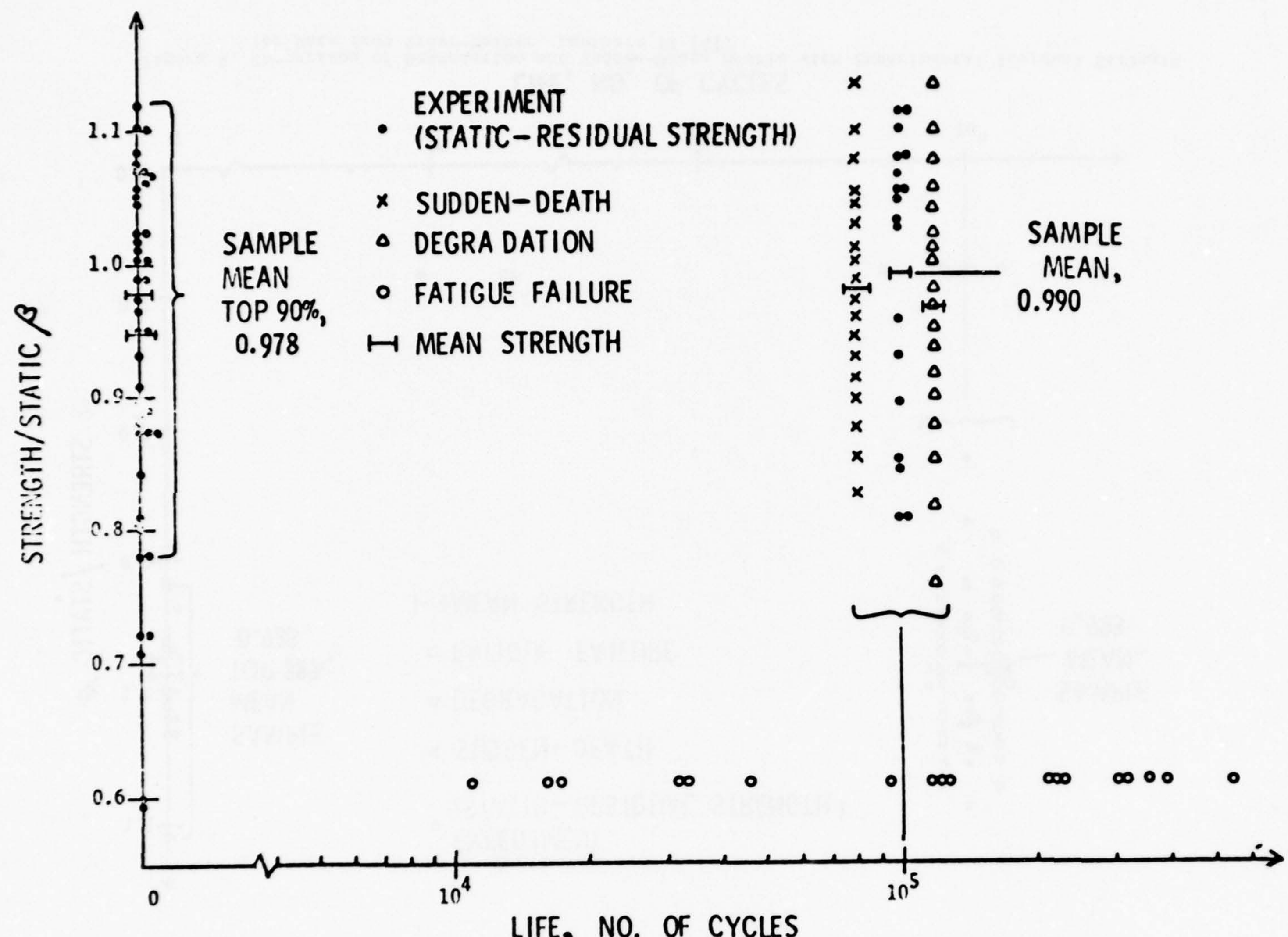


Figure 3. Comparison of Degradation and Sudden-Death Models with Experimental Residual Strength for Data from Awerbuch-Hahn [5].

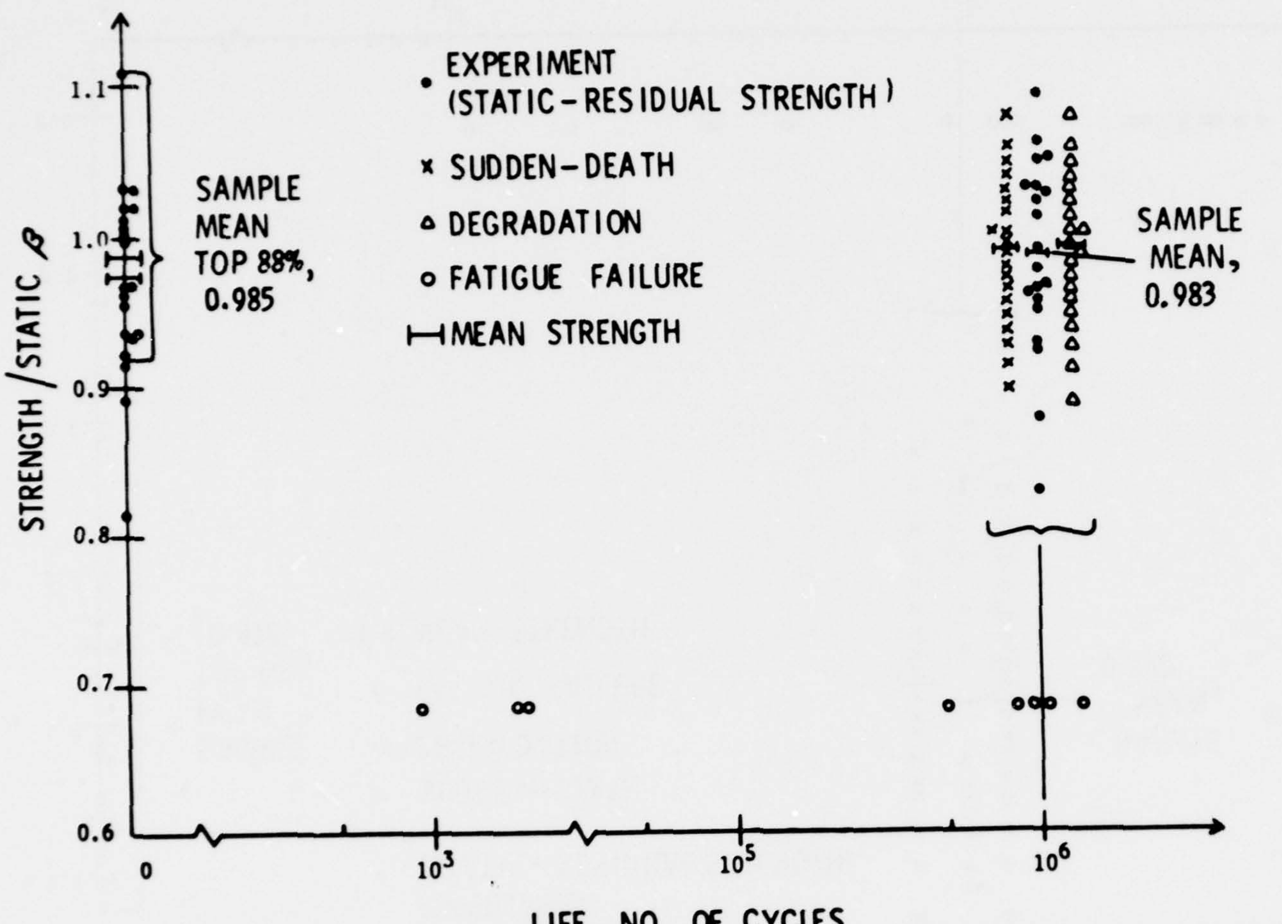


Figure 4. Comparison of Degradation and Sudden-Death Models with Experimental Residual Strength for Data from Ryder-Walker, Laminate II [4].

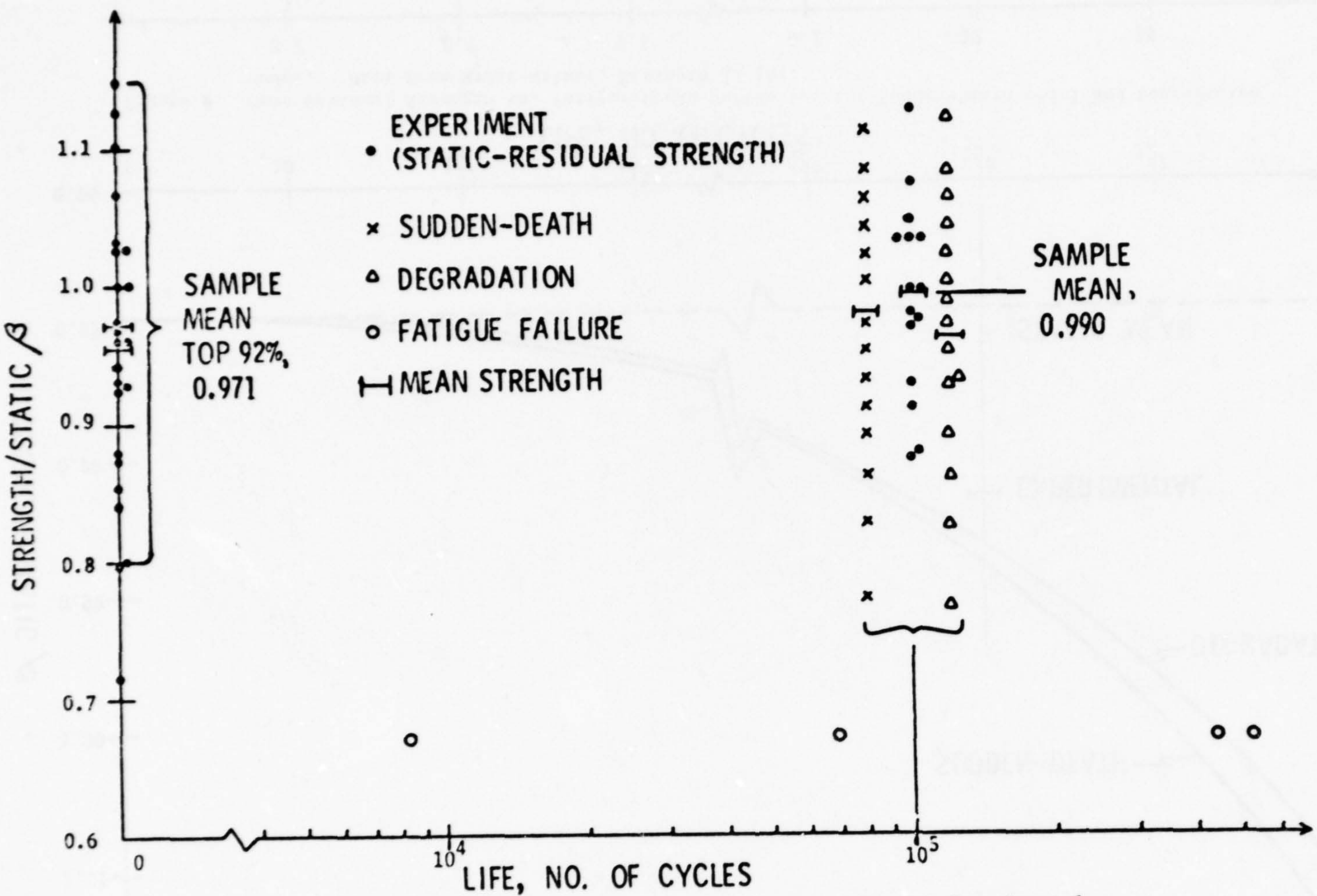


Figure 5. Comparison of Degradation and Sudden-Death Models with Experimental Residual Strength for Data from Wang [6].

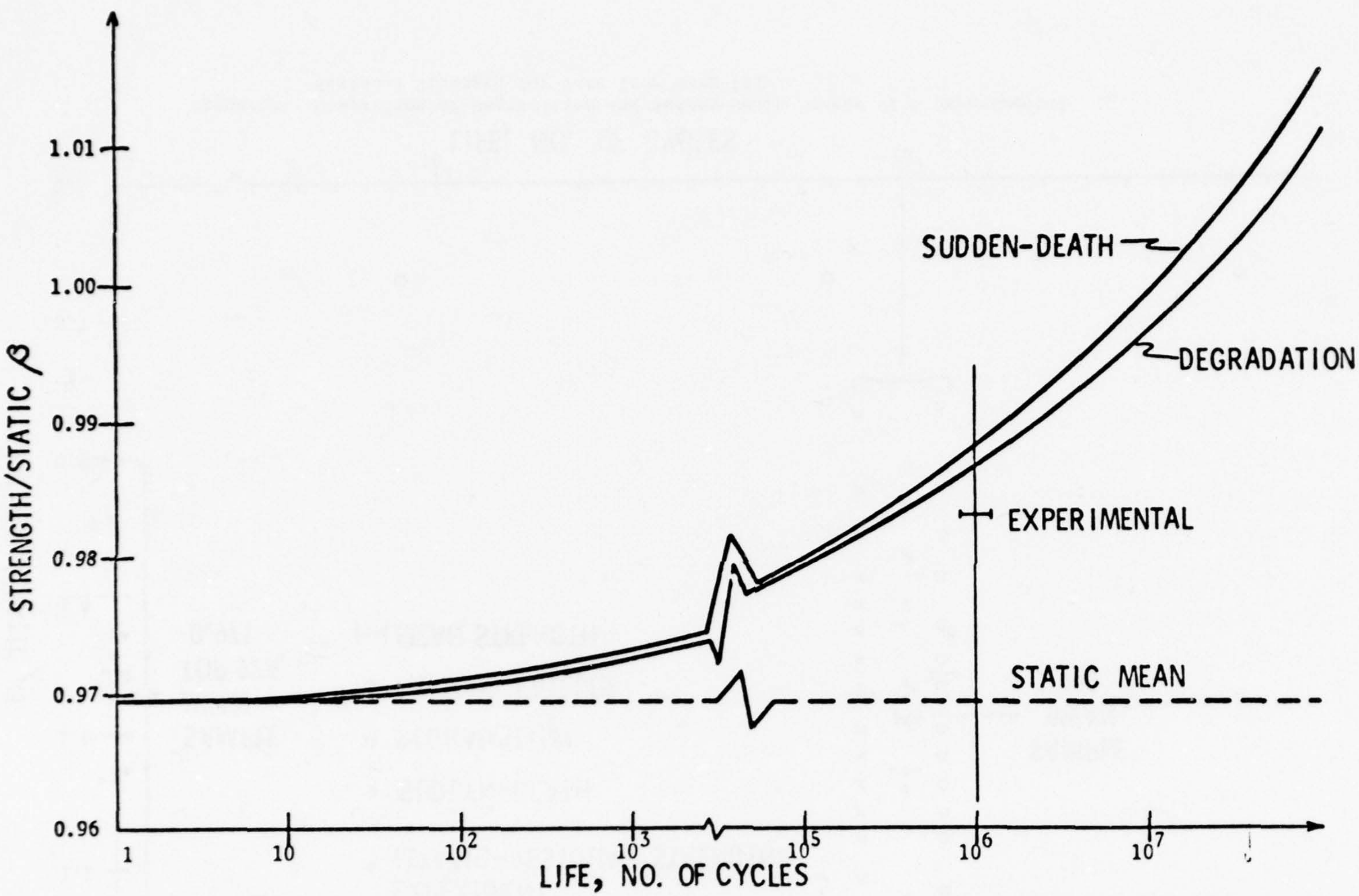
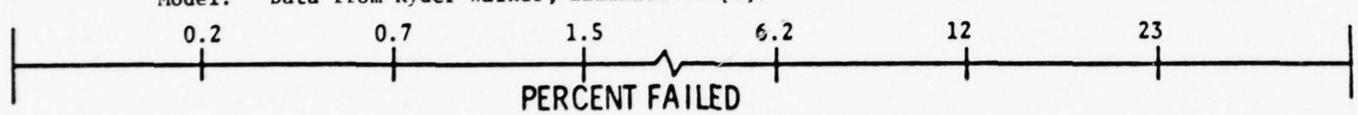


Figure 6. Mean Residual Strength vs. Fatigue Cycle Curves for the Sudden-Death Model and Degradation Model. Data from Ryder-Walker, Laminate II [4].



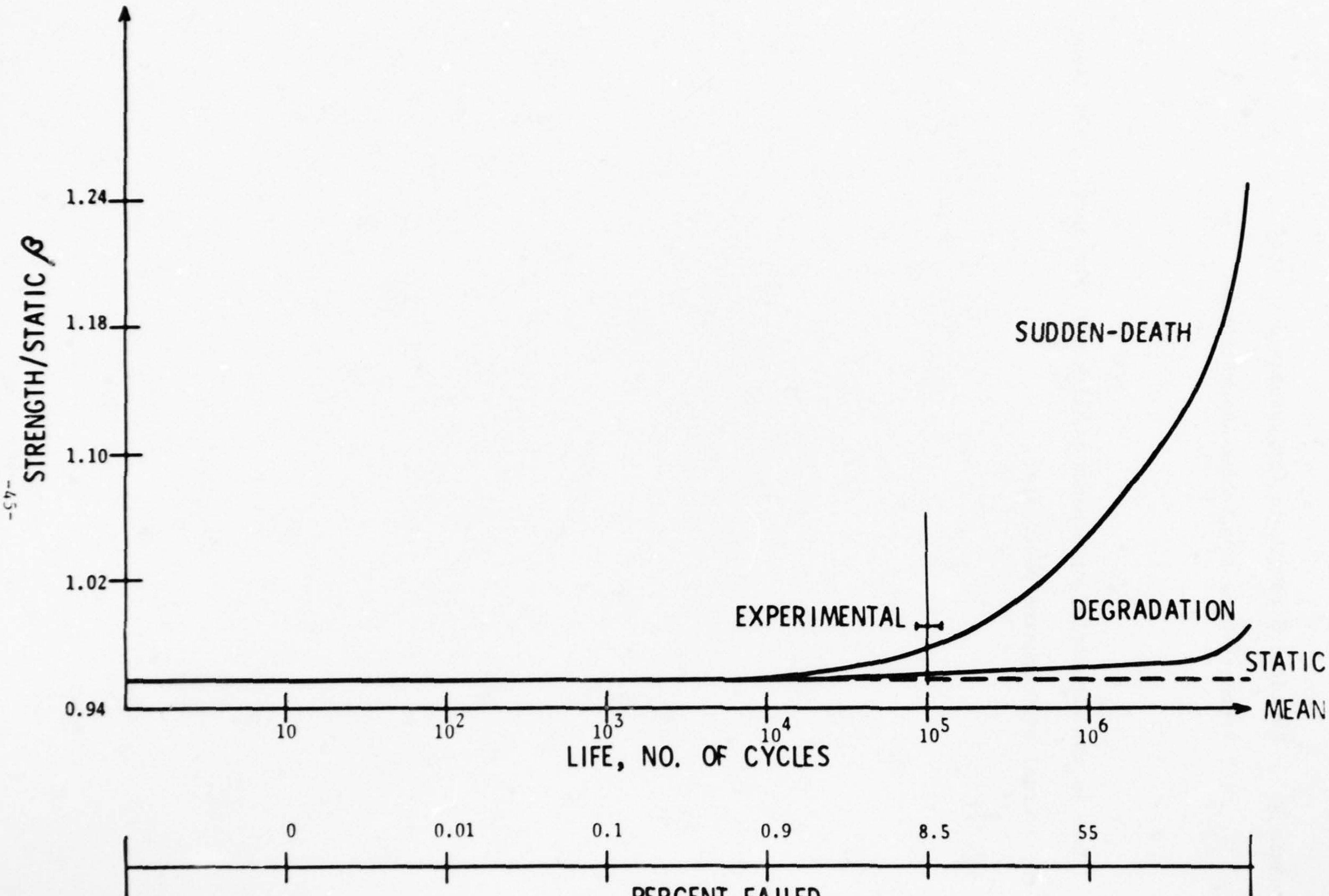


Figure 7. Mean Residual Strength vs. Fatigue Cycle Curves for the Sudden-Death Model and Degradation Model. Data from Wang [6].

Appendix B. Residual Strength in Fatigue Based on the Strength-Life Equal Rank Assumption

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Residual Strength in Fatigue Based on the
Strength-Life Equal Rank Assumption*

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Abstract

Equations for the distribution of the residual strength after fatigue are derived. They are compared with experimental data for a few graphite/epoxy composites. The theory is based on an assumption first introduced by Hahn and Kim, which states that for a given specimen its rank in static strength is equal to its rank in fatigue life, or the strength-life equal rank assumption. The equations contain a free parameter which is more versatile in matching experimental data. It is found that two processes are present during fatigue, one degradates individual specimens and tends to lower the mean strength, the other is the weeding out of weak specimens by fatigue failure which tends to increase the residual mean strength. The change of residual strength can be of weak degradation, strong degradation, or increase in strength. Among the test data studied, two have weak degradation, two have strong degradation and two have increase in strength.

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Residual Strength in Fatigue Based on the Strength-Life Equal Rank Assumption

Scope

In studying the residual strength of composite materials under fatigue loading, Halpin et al [1] proposed a degradation equation that is based on the crack propagation of homogeneous materials. Realizing that fatigue failure of composites is not dictated by the initiation and growth of a dominant crack, Hahn and Kim [2] introduced the concept of rate of change of residual strength, without referring to any crack. They assumed the time rate of decrease of residual strength is inversely proportional to the residual strength to a certain power. From this deterministic residual strength equation, and the static strength distribution, they derived the fatigue life distribution. Following the same approach as Hahn and Kim, Yang and Liu [3] further derived the residual strength distribution, and compared the results with one group of experimental data.

In a recent paper, Chou and Croman [4] demonstrated that the degradation model used by Hahn and Yang is overly restrictive. According to that model, once the static strength and the fatigue life distributions are known, the residual strength at all fatigue cycles are fixed, there is no open parameter to accommodate different residual strength distributions. In this paper, we shall introduce a degradation equation which contains an additional parameter. This parameter can be adjusted to fit various residual strength data for a material of fixed static strength and life distributions.

Hahn and Kim [2] have introduced the assumption of a unique relation between the ranks in static strength and in fatigue life of a given specimen. We shall call this the strength-life equal rank assumption. This is a very fundamental assumption because if this is not true, then the equation of degradation of residual strength of a specific specimen can not be deterministic, and random variables must be used. In [4], it was shown that the deterministic degradation equation, used by Yang and Liu implies this equal-rank assumption. In fact, all deterministic fracture mechanics equations in fatigue all imply this equal rank assumption.

In this paper, we shall first make the strength-life equal rank assumption. Based on this assumption, the constraints on the degradation equation are derived. Then a possible form of degradation equation is introduced. The residual strength distribution is then derived, and compared with existing experimental results.

Another concept introduced here concerns two processes going on simultaneously during fatigue. One is the degradation of individual specimens, the other is the weeding out of weak specimens by fatigue failure. Depending on which of these two processes is more dominant, the mean of the residual strength can be smaller or larger than the mean of the static strength. The former is called strong degradation, the latter weak degradation.

Residual Strength Equations

Let us assume that both the static strength and the fatigue life distributions are of the two-parameter Weibull form, and the parameters are known from experimental data. Let $R(0)$ be the random variable of static strength and x be its value, then the distribution function of the static strength is

$$F_{R(0)}(x) = P[R(0) \leq x] = 1 - \exp(-x^{\alpha}) \tag{1}$$

where α is the shape parameter. The strength x has been normalized by its characteristic value (scale parameter), or

$$x = \frac{\text{static strength}}{\beta} \tag{2}$$

where β is the scale parameter of static strength.

In order to compare the static strength with the fatigue life at a stress S, we formulate the distribution of strength for those specimens that have strength larger than S. This is accomplished by taking the conditional probability,

$$\begin{aligned} F_{R(0),S}(x) &= P[R(0) \leq x \mid R(0) > S] \\ &= 1 - \exp(-x^{\alpha} + S^{\alpha}) \end{aligned} \tag{3}$$

Let N be the random variable of fatigue life, and n its value, then

$$F_N(n) = P(N \leq n) = 1 - \exp(-n^{\alpha_1}) \tag{4}$$

where α_1 is the shape parameter. The life n has been normalized by the characteristic life (scale parameter), or

$$n = \frac{\text{life}}{n_0} \tag{5}$$

where n_0 is the scale parameter of the life distribution. In this paper, all stress and strength will be normalized by β, and life by n_0.

The assumption that for a given specimen the static strength and fatigue life have the same rank is then represented by

$$F_{R(0),S}(x_\gamma) = F_N(n_\gamma) \tag{6}$$

where x_γ is a value that gives a cumulative distribution of static strength of $1-\gamma$, or, 100γ percent of the specimens has strength higher than x_γ.

$$F_{R(0),S}(x_\gamma) = 1 - \gamma \tag{7}$$

and, similarly, n_γ is the life that gives a cumulative distribution of $1-\gamma$, or

$$F_N(n_\gamma) = 1 - \gamma. \tag{8}$$

It follows then,

$$n_\gamma^{\alpha_1} = x_\gamma^\alpha - S^\alpha \tag{9}$$

This equation states that the particular specimen that has a static strength x_γ, will have a fatigue life n_γ, and x_γ and n_γ are related by Eq. (9).

Next we shall propose a family of degradation equations that is compatible with the equal rank relation, Eq. (9). There will be no loss of generality by considering that the residual strength is S when fatigue failure occurs.

In writing a degradation equation, we follow a specific specimen with a static strength x_γ, and ask for its residual strength as a function of time, or cycle. Let the residual strength be y, which is a function of x_γ and n. Then, the degradation equation

$$y = y(x_\gamma, n) \tag{10}$$

must satisfy the condition that it passes through the points $(x_\gamma, 0)$ and (S, n_γ), or

$$\begin{aligned} y(x_\gamma, 0) &= x_\gamma \\ y(x_\gamma, n_\gamma) &= S \end{aligned} \tag{11}$$

where n_γ is related to x_γ by Eq. (9).

It can be shown easily that Eqs. (11) are satisfied by the equation

$$\frac{x_\gamma^\alpha - y^\alpha}{x_\gamma^\alpha - S^\alpha} = \left(\frac{n^{\alpha_1}}{x_\gamma^\alpha - S^\alpha}\right)^i \tag{12}$$

where i is a constant. This can be written as

$$y^\alpha = y_\gamma^\alpha - (x_\gamma^\alpha - S^\alpha)^{1-i}(n^{\alpha_1})^i = x_\gamma^\alpha - n_\gamma^{\alpha_1}\left(\frac{n}{n_\gamma}\right)^{i\alpha_1} \tag{13}$$

The corresponding degradation rate is

$$\frac{dy}{dn} = -\frac{i\alpha_1}{\alpha}\,\frac{n^{i\alpha_1 - 1}\,(x_\gamma^\alpha - S^\alpha)^{1-i}}{y^{\alpha-1}} \tag{14}$$

By assuming different values to i, a family of degradation curves can be constructed. In Figs. 1 and 2, a family of y vs. n curves are shown, each for a particular set of values of α and α_1. The sudden death curves as discussed in [4], as well as the Hahn-Yang model, are also shown in these figures. All curves are for the specimen that has a value of x_γ equal to 0.9.

Next let us derive the distribution for the residual strength after n_{γ_1} cycles of fatigue, when 100 (γ_1) percent of the specimens have survived. Fig. 3 shows schematically the residual strength as a function of n for two values of x. For $n = n_{\gamma_1}$, Eq. (13) becomes

$$y^\alpha = x_\gamma^\alpha - (x_\gamma^\alpha - S^\alpha)^{1-i}\,(n_{\gamma_1}^{i\alpha_1}) = x_\gamma^\alpha - n_\gamma^{\alpha_1}\left(\frac{n_{\gamma_1}}{n_\gamma}\right)^{i\alpha_1} \tag{15}$$

To obtain the distribution of $R(n_{\gamma_1})$, let us first find the distribution of the top γ_1-percent of the static strength, or

$$F_{R(0),\gamma_1}(x_\gamma) = 1 - \exp[-\,x_\gamma^\alpha + x_{\gamma_1}^\alpha] \tag{16}$$

The corresponding density function is

$$f_{R(0),\gamma_1}(x_\gamma) = \alpha x_\gamma^{\alpha-1} \exp[- x_\gamma^\alpha + x_{\gamma_1}^\alpha] \tag{17}$$

The derivative dx_γ/dy is

$$\frac{dx_\gamma}{dy} = \frac{(y/x_\gamma)^{\alpha-1}}{[1 - (1-i)n_{\gamma_1}^{\alpha_1 i}(x_\gamma^\alpha - S^\alpha)^{-i}]} \tag{18}$$

The density function of the residual strength is then

$$f_{R(n_{\gamma_1})}(y) = f_{R(0),\gamma_1}(x_\gamma)\frac{dx_\gamma}{dy}$$

$$= \frac{\alpha\, y^{\alpha-1} \exp(- x_\gamma^\alpha + x_{\gamma_1}^\alpha)}{[1 - (1-i)n_{\gamma_1}^{\alpha_1 i}(x_\gamma^\alpha - S^\alpha)^{-i}]} \tag{19}$$

Due to the complexity of Eq. (15), x_γ cannot be expressed explicitly as a function of y. It is understood that in Eq. (19), all x_γ terms are to be considered as functions of y, and the relation $x_\gamma = x_\gamma(y)$ is implicit from Eq. (15). The cumulative distribution function of the residual strength is

$$F_{R(n_{\gamma_1})}(y) = \int_S^y f_{R(n_{\gamma_1})}(y)\, dy \tag{20}$$

The distribution $F_{R(n_{\gamma_1})}(y)$ may be obtained numerically by considering

$$F_{R(n_{\gamma_1})}(y) = \int_{x_{\gamma_1}}^{x_\gamma} f_{R(0),\gamma_1}(x_\gamma)dx_\gamma$$

$$= 1 - \exp[- x_\gamma^\alpha(y) + x_{\gamma_1}^\alpha] \tag{21}$$

where $x_\gamma(y)$ indicates that x_γ is a function of y.

For values of x_γ from x_{γ_1} to ∞, the corresponding values for y are calculated from Eq. (15), and the values of $F_{R(n_{\gamma_1})}(y)$ from Eq. (21).

The mean of the residual strength is

$$\mu_{R(n_{\gamma_1})} = \int_S^\infty y\, f_{R(n_{\gamma_1})}(y)dy \tag{22}$$

which can be expressed as

$$\mu_{R(n_{\gamma_1})} = \int_{x_{\gamma_1}}^\infty \left[x_\gamma^\alpha - (x_\gamma^\alpha - S^\alpha)^{1-i}(n_{\gamma_1}^{i\alpha_1})\right]^{1/\alpha} \alpha x_\gamma^{\alpha-1} \exp\left[- x_\gamma^\alpha + x_{\gamma_1}^\alpha\right] dx_\gamma \tag{23}$$

This can be integrated numerically.

It is interesting to note that for the special case of i = 1, explicit expressions for the residual strength distribution can be obtained. For i = 1, Eqs. (15) and (14) reduce to

$$y^\alpha = x_\gamma^\alpha - n_{\gamma_1}^{\alpha_1} \tag{24}$$

$$\frac{dy}{dn} = - \frac{\alpha_1}{\alpha} \frac{n^{\alpha_1 - 1}}{y^{\alpha-1}} \tag{25}$$

From Eqs. (19) and (24), we have

$$f_{R(n_{\gamma_1})}(y) = \alpha y^{\alpha-1} \exp[- y^\alpha + S^\alpha] \tag{26}$$

This is exactly the same density function as $f_{R(0),S}(x)$, or, if we use y for both static and residual strength,

$$f_{R(n_{\gamma_1})}(y) = f_{R(0),S}(y), \quad \text{for } i = 1 \tag{27}$$

Note that Eq. (26) is independent of n; the residual strength distribution in this case is always equal to its static distribution; the mean of the residual strength is equal to the static mean for all values of n. As will be explained further later, for this case, the process of weeding out of weak specimens in fatigue just balances the degradation process of each surviving specimen.

Strong Degradation and Weak Degradation

Let us now discuss the degradation model represented by Eqs. (13) and (14). During fatigue, two processes are going on simultaneously: one is the degradation of the individual specimen, the other is the weeding out of weak specimens by fatigue failure.

For an individual specimen, the residual strength always decreases according to Eqs. (13) and (14). At a given fatigue life, the surviving specimens all have a lower strength than their respective static strength. Therefore, the mean of the residual strength is always smaller than the mean of the static strength of <u>these specimens</u>. Since γ_1-percent of the specimens have survived at the life n_{γ_1}, the mean of the residual strength is always smaller than the mean of the static top γ_1-percent, or

$$\mu_{R(n_{\gamma_1})} < \mu_{R(0),\gamma_1} \quad . \tag{28}$$

On the other hand, the weeding out process eliminates the weak specimens; comparing with the original population, the surviving specimens have a larger mean static strength than the mean static strength of the total population, or

$$\mu_{R(0),\gamma_1} > \mu_{R(0),S} \tag{29}$$

In other words, the weeding out process tends to increase the mean strength of the surviving specimens. In practical application, we may want to compare

the mean of the residual strength with the mean of static strength of the total population. If $\mu_{R(n_{\gamma_1})}$ is larger than $\mu_{R(0),S}$, we may say the average residual strength has increased. If $\mu_{R(n_{\gamma_1})}$ is less than $\mu_{R(0),S}$, the average residual strength has decreased. Figure 4 shows a plot of the various mean strengths as a function of fatigue life n, for a particular set of values of α and α_1.

It can be seen from Eq. (27) that for the case of i = 1, the mean of the residual is equal to the mean of static total population, which is demonstrated by a straight horizontal line in Fig. 4. It can be shown that for $i \geq 1$

$$\mu_{R(n_{\gamma_1})} \geq \mu_{R(0),S}, \quad i \geq 1 \tag{30}$$

We shall call this case the weak degradation.

For the case of $i \leq 1$,

$$\mu_{R(n_{\gamma_1})} \leq \mu_{R(0),S}, \quad i \leq 1 \tag{31}$$

which shall be called the strong degradation. Both types of degradation have been observed experimentally for composite materials.

When i approaches infinity, it can be seen from Eq. (13) that $y \to x_{\gamma_1}$ for $n < n_{\gamma_1}$, and $y = S$ for $n = n_{\gamma_1}$; which indicates that the residual strength of a specimen is equal to its static strength right up to the fatigue failure. This is exactly the definition of the sudden-death model discussed in [4]. In Fig. 4 the region above the sudden death line represents an increase in residual strength in each specimen.

Analysis of Experimental Sample Data

It is often desirable to make simple calculations and draw conclusions from experimental data, without going through rigorous fitting of distribution functions and estimation of parameters. We shall show in this section that the nature of the degradation can be determined directly from test data.

First let us calculate the percent failure at a given fatigue life. If no fatigue tests are conducted after n_1 when all surviving specimens are used for residual strength measurement, the percent failed is simply the number of failed specimens divided by the total number of specimens. For generality, we shall discuss the case where residual strength is measured at more than one life, and fatigue tests at life longer than the residual strength life are conducted. This is represented schematically in Fig. 4. Let f_1 be the number of specimens failed before n_1, r_1 the number of specimens tested for residual strength at life n_1, and f_2, f_3 and r_2 are similarly defined as shown. The total number of specimens used for fatigue and residual strength tests is t, where

$$t = f_1 + f_2 + f_3 + r_1 + r_2. \tag{32}$$

The percent failed up to n_1 is then

$$\begin{aligned}(1-\gamma_1) &= \text{percent failed before } n_1 \\ &= f_1/t \end{aligned} \tag{33}$$

Among those surviving n_1 cycles, we shall censor r_1 specimen randomly, and treat the remaining specimens as representative of the surviving population. Among those that survived n_1, the percent failed between n_1 and n_2 is $f_2/(f_2 + f_3 + r_2)$. The corresponding percent of total population that would fail before n_2 is then

$$\begin{aligned}(1-\gamma_2) &= \text{percent failed before } n_2 \\ &= \left(1 - \frac{f_1}{t}\right)\left(\frac{f_2}{f_2+f_3+r_2}\right) + \frac{f_1}{t}\end{aligned} \tag{34}$$

Let r_0 be the number of specimens used for static strength measurement, and these strength values are arranged in increasing order

$$x_1 < x_2 < x_3 \cdots < x_{r_0} \tag{35}$$

where x_1 is the smallest strength above S. Strength points below S will not be used. The sample static mean is

$$\bar{\mu}_{R(0),S} = \frac{x_1 + x_2 + \cdots x_{r_0}}{r_0} \tag{36}$$

The top γ_1-percent of the specimen includes ν specimens from the top, where

$$\nu_1 \cong \gamma_1 \; r_0 \tag{37}$$

Since $\gamma_1 r_0$ is most likely not an integer, ν_1 can be taken as the integer closest to it. The top γ_1-percent sample mean is

$$\bar{\mu}_{R(0),\gamma_1} = \frac{x_{(r_0-\nu_1)} + x_{(r_0-\nu_1+1)} + \cdots + x_{r_0}}{\nu_1} \tag{38}$$

Similarly,

$$\nu_2 \cong \gamma_2 \; r_0 \tag{39}$$

and

$$\bar{\mu}_{R(0),\gamma_2} = \frac{x_{r_0-\nu_2} + x_{r_0-\nu_2+1} + \cdots x_{r_0}}{\nu_2} \tag{40}$$

The sample mean of the residual strength is simply the average of the r_1 and r_2 data points, respectively, or

$$\bar{\mu}_{R(n_{\gamma_1})} = \frac{y_1 + y_2 + y_3 + \cdots y_{r_1}}{r_1} \tag{41}$$

where

$$y_1 < y_2 < y_3 \cdots < y_{r_1}$$

are the residual strength data points at n_1. If

$$\bar{\mu}_{R(n_{\gamma_1})} > \bar{\mu}_{R(0),\gamma_1} \tag{42}$$

there is an increase in residual strength. If

$$\mu_{R(0),S} < \bar{\mu}_{R(n_{\gamma_1})} < \bar{\mu}_{R(0),\gamma_1} \tag{43}$$

it belongs to the weak degradation type. If

$$\bar{\mu}_{R(n_{\gamma_1})} < \bar{\mu}_{R(0),S} \tag{44}$$

it belongs to the strong degradation type.

Comparison with Experimental Data

Six sets of residual strength test data for graphite/epoxy composites under tension fatigue loading will be used for comparison. These data were taken from a few sources, as discussed in [4]. The sample means and the estimated Weibull parameters of the static strength and fatigue life of these test data are reprinted in Table I for easy reference.

Table II list the sample data of these five sets, including the cycle when residual strength is measured ($n_{\gamma 1}$), percent failed ($1-\gamma_1$), sudden-death mean ($\bar{\mu}_{R(0),\gamma_1}$) static sample mean ($\bar{\mu}_{R(0),S}$), and the residual sample mean ($\bar{\mu}_{R(n_{\gamma_1})}$). According to Eqs. 42 to 44, two of these five cases belong to strong degradation, two are weak degradation, and two have increase in residual strength.

The mean residual strength as calculated from Eq. (23) has been plotted in Figs. 4 and 6-9 for five pairs of α and α_1 which were picked to correspond with the five sets of data. Here the mean strength has been presented as a function of fatigue life, n_γ and the parameter i. Directly below the abscissa are the percentage of specimens failed at selected values of n_γ. Curves calculated from Eq. (23) are given for values of i leading to both strong and weak degradation as well as i = 1. The curve resulting from i = ∞ is also labeled as the sudden-death model. In addition the Hahn-Yang degradation model is presented in each plot. The sample means of the experimental residual strength data are plotted at their respective fatigue lives as horizontal bars.

Fig. 4 is plotted for a static strength shape parameter α of 15.3 and a fatigue life shape parameter α_1 of 1.18 corresponding to Yang-Liu data. The mean residual strength is seen to be in the strong degradation region. The curve which best approximates this sample mean point is obtained from the present approach with i between 0.3 and 0.4.

The values of α and α_1 used in Fig. 6 are 23.9 and 1.31 respectively, corresponding to the Ryder-Walker Laminate I data. The experimental residual mean is in the strong degradation region and can be fitted by either the Hahn-Yang degradation model or Eq. (23) with i = 0.5.

Fig. 7 is based on $\alpha = 16.8$ and $\alpha_1 = 0.30$, corresponding to the Ryder-Walker Laminate II data. The sample residual mean falls in the weak degradation region of this plot. The present approach with $i = 3$ passes through this sample mean. However the sudden death and Hahn-Yang curves are not too far off either.

Values of $\alpha = 9.7$ and $\alpha_1 = 0.75$ were used to construct Fig. 8. The Awerbuch-Hahn data gives two sample residual strength means and these are plotted at their respective fatigue lives. The first is in the increase of strength region while the other one falls into the weak degradation region. None of the approaches considered so far is capable of handling an increase of residual strength.

The curves in Fig. 9 are calculated with $\alpha = 10.2$ and $\alpha_1 = 0.94$. The residual strength sample mean of the Wang data lies above the sudden death curve or in the increase of strength region. Once again the existing models are incapable of handling this phenomenon.

It is interesting to note that the Yang-Liu data and Ryder-Walker Laminate I data sample means are both in the strong degradation region. Both of these are for graphite/epoxy specimens with 25% 0° plies. The Ryder-Walker Laminate II data specimens have 67% 0° plies and the sample residual mean lies in the weak degradation region. The Awerbuch-Hahn and Wang data sets are for specimens of 100% 0° plies. Here we observe mean residual strengths in the increase of strength region. These few observations tend to suggest that the percentage of 0° plies has a strong influence in the residual strength.

For the two cases of 100% 0° plies, there is an increase of residual strength. The data is not sufficient for a firm conclusion. From fracture mechanics point of view, it is possible for a specimen to have an increase in residual strength, during the early stage of fatigue as discussed by Reifsnider, et al. [8].

Eq. (12) cannot accommodate any increase in residual strength. It can be shown that Eq. (12) can be generalized by adding another parameter, so that an initial increase followed by later decrease of residual strength can be obtained.

Conclusions

1. Based on the strength life equal rank assumption, a particular degradation equation is introduced. This equation contains an open parameter which can be adjusted to fit various test results. There is not enough test data to ascertain if the proposed equation is the best suited.

2. The degradation equation proposed here cannot accommodate initial increase in residual strength, which has been observed in a few cases of fatigue of composite materials. Based on the present assumption and approach, a more general equation can be selected easily.

3. There are two basic processes acting during fatigue, one is the degradation of individual specimens which tends to lower the mean residual strength. The other is the weeding out of weak specimens by fatigue failure, which tends to increase the mean residual strength. If the former process is predominant, the residual mean is lower than the static mean of the total population, and the degradation is strong. If the latter is predominant, the residual mean is higher than the static mean, and the degradation is weak.

4. For the test data on graphite/epoxy studied here, two sets show increase in residual strength, two show weak degradation, and two show strong degradation.

Nomenclature

$f_Z(z)$	Probability density function for random variable Z.
f_j	Number of fatigue failed specimens. $(j = 1,2,\cdots)$
$F_Z(z)$	Cumulative distribution function for random variable Z.
i	As exponent, the open parameter in the residual strength equation.
n	Value of fatigue life.
n_0	Weibull scale parameter for fatigue life or characteristic life.
n_γ	Value that gives cumulative distribution of fatigue life of $1-\gamma$.
n_{γ_1}	Value of fatigue life at which residual strength test is performed.
N	Random variable of fatigue life.
$P(Z\leq z)$	Cumulative distribution function for random variable Z.
r_0	Number of specimens failed by static strength test.
r_j	Number of specimens failed by residual strength test. $(j = 1,2,\cdots)$
$R(0)$	Random variable of static strength.
$R(n_{\gamma_1})$	Random variable of residual strength.
S	Maximum stress applied in fatigue cycling.
t	Total number of specimens used for fatigue and residual strength tests.
x	Value of static strength.
x_γ	Value that gives cumulative distribution of static strength of $1-\gamma$.
x_{γ_1}	Value of static strength that corresponds to n_{γ_1}.
y	Value of residual strength.
α	Weibull shape parameter for static strength.
α_1	Weibull shape parameter for fatigue life.
β	Weibull scale parameter for static strength or characteristic strength.
γ	Percent of specimens surviving.
μ	Mean.
$\bar{\mu}$	Sample mean.
ν_j	Number of specimens in top γ_j-percent of specimens to be subjected to static strength test. $(j = 1,2,\cdots)$

References

1. Halpin, J.C., Jerina, K.L., and Johnson, T.A., "Characterization of Composites for the Purpose of Reliability Prediction," Analysis of Test Methods for High Modulus Fibers and Composites, ASTM STP 521, American Society for Testing and Materials, 1973, p. 5.

2. Hahn, H.T. and Kim, R.Y., "Proof Testing of Composite Materials," Jour. Composite Materials, Vol. 9, No. 3, July, 1975, pp. 297-311.

3. Yang, Jann N. and Liu, M.D., "Residual Strength Degradation Model and Theory of Periodic Proof Tests for Graphite/Epoxy Laminates," AFML-TR-76-225, Wright-Patterson AFB, 1976.

4. Chou, P.C. and Croman, R., "Degradation and Sudden-Death Models of Fatigue of Graphite/Epoxy Composites," presented at the ASTM Symposium on Composite Materials; Testing and Design, New Orleans, La., March 20, 1978.

5. Ryder, J.T. and Walker, E.K., "Ascertainment of the Effect of Compressive Loading on the Fatigue Lifetime of Graphite Epoxy Laminates for Structural Application," AFML-TR-76-241, Wright-Patterson AFB, 1976.

6. Awerbuch, J. and Hahn, H.T., "Fatigue and Proof-Testing of Unidirectional Graphite/Epoxy Composite," in Fatigue of Filamentary Composite Materials, STP 636, American Soc. for Testing and Materials, 1977, pp. 248-266.

7. Wang, A.S.D., private communication, Drexel University test data.

8. Reifsnider, K.L., Stinchcomb, W.W., and O'Brien, T.K., "Frequency Effects on a Stiffness-Based Fatigue Failure Criterion in Flawed Composite Specimens," in Fatigue of Filamentary Composite Materials, STP 636, American Soc. for Testing and Materials, 1977, pp. 171-184.

TABLE I Weibull Parameters of the Graphite/Epoxy Composites Used for Comparison, Ref. [4]

Data Source [Ref.]	Static Strength				Fatigue Life		
	Shape Parameter α	Scale Parameter, β ksi (GPa)	Sample Mean $\bar{x}/\beta$	Weibull Mean μ/β	Shape Parameter α_1	Scale Parameter n_0 cycles	Ratio of Shape Parameters, $c = \alpha/\alpha_1$
Yang-Liu [3]	15.3	78.1 (0.54)	0.965	0.966	1.18	1.65×10^5	13.0
Ryder-Walker Laminate I [5]	23.9	70.7 (0.49)	0.979	0.978	1.31	1.28×10^5	18.2
Ryder-Walker Laminate II [5]	16.8	145.9 (1.01)	0.971	0.969	0.30	8.54×10^8	56.1
Awerbuch-Hahn [6]	9.7	207.5 (1.43)	0.947	0.950	0.75	1.92×10^6	12.9
Wang [7]	10.2	222.3 (1.53)	0.954	0.952	0.94	1.27×10^6	10.8

TABLE II Determination of the Type of Degradation from Sample Data, Eqs. 42 to 44.

Data Source [Ref.]	n_{γ_1} Cycles	γ_1 (percent survived at n_{γ_1})	Static top γ_1-percent mean = Sudden death mean $(\bar{\mu}_{R(0),\gamma_1})$	Static mean of total population $\bar{\mu}_{R(0),S}$	Residual Mean $\bar{\mu}_{R(n_{\gamma_1})}$	Type of Degradation
Yang Liu [3]	26,000	82	0.993	0.965	0.928	strong degradation
Ryder-Walker Laminate I [5]	31,400	91	0.986	0.979	0.963	strong degradation
Ryder-Walker Laminate II [5]	1×10^6	88	0.985	0.971	0.983	weak degradation
Awerbuch-Hahn [6]	1×10^5	90	0.978	0.947	0.990	increase in strength
	5×10^5	71	1.017		0.962	weak degradation
Wang [7]	1×10^5	92	0.971	0.954	0.990	increase in strength

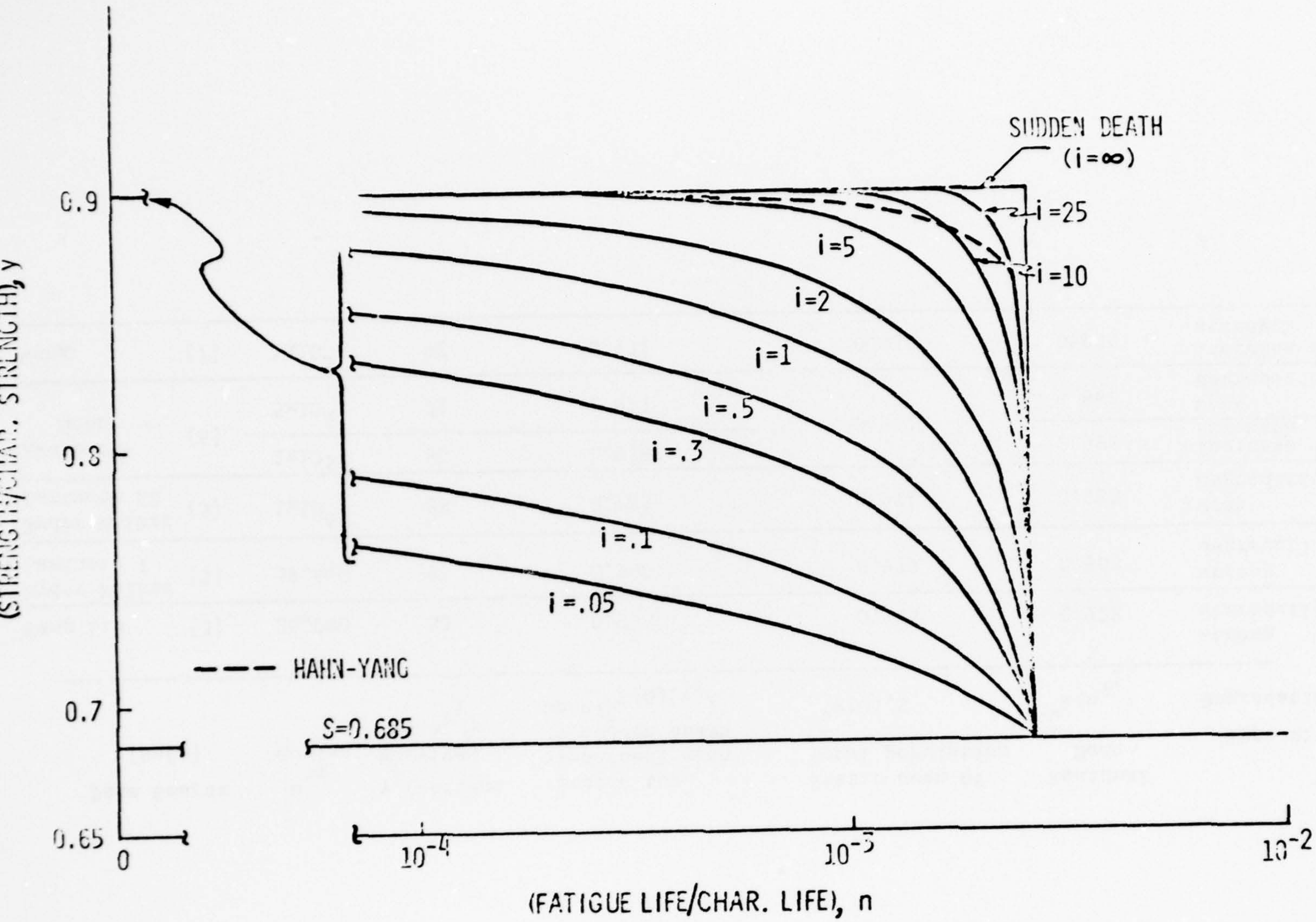

Figure 1. Residual Strength vs. Fatigue Life Curves for a Specimen with Initial Strength of 0.9 β, with Different Values of the Parameter i. Static Strength Shape Parameter α = 16.8, Life Shape Parameter α_1 = 0.30.

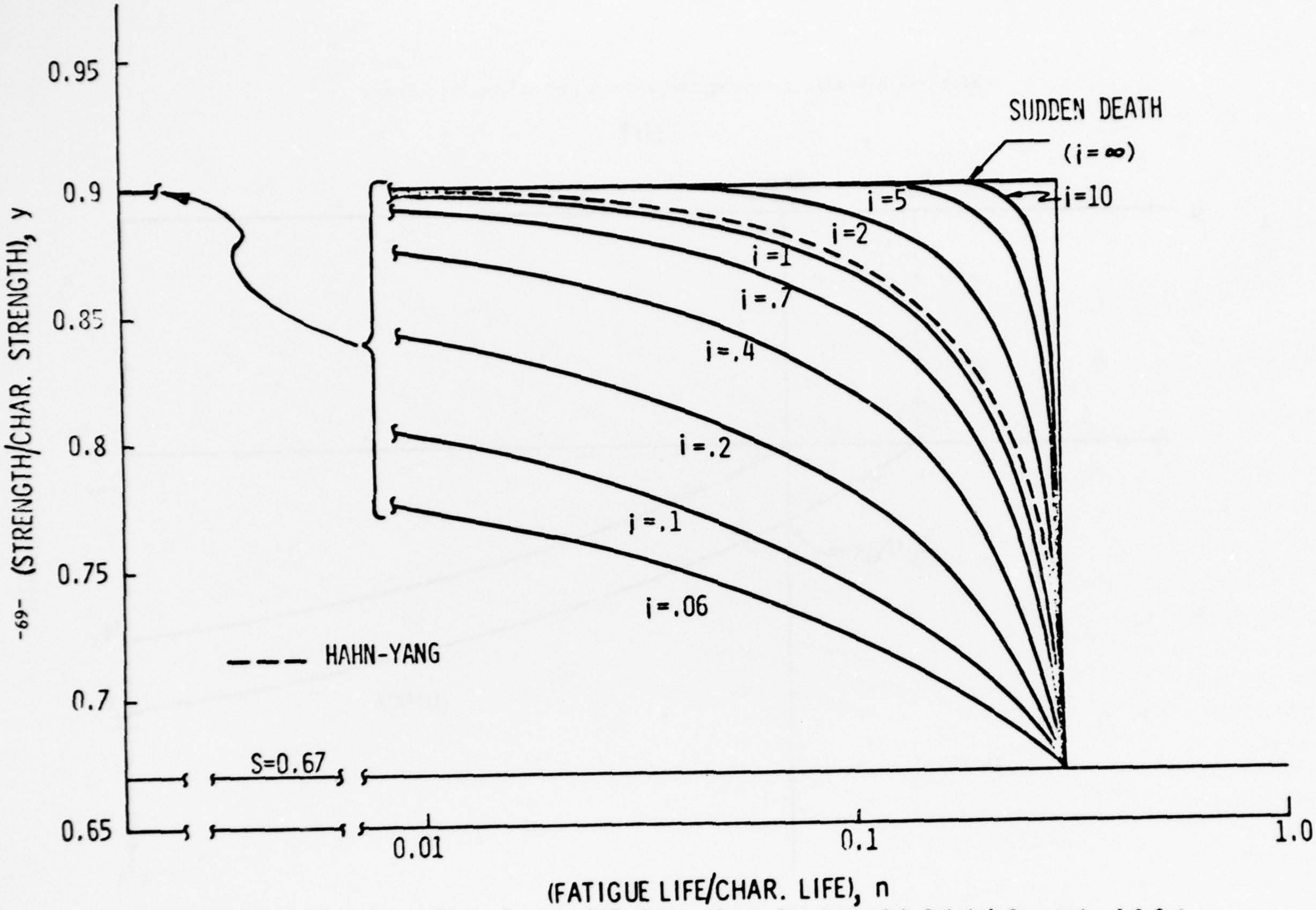

Figure 2. Residual Strength vs. Fatigue Life Curves for a Specimen with Initial Strength of 0.9 β with Different Values of the Parameter i. Static Strength Shape Parameter $\alpha = 10.2$, Life Shape Parameter $\alpha_1 = 0.94$.

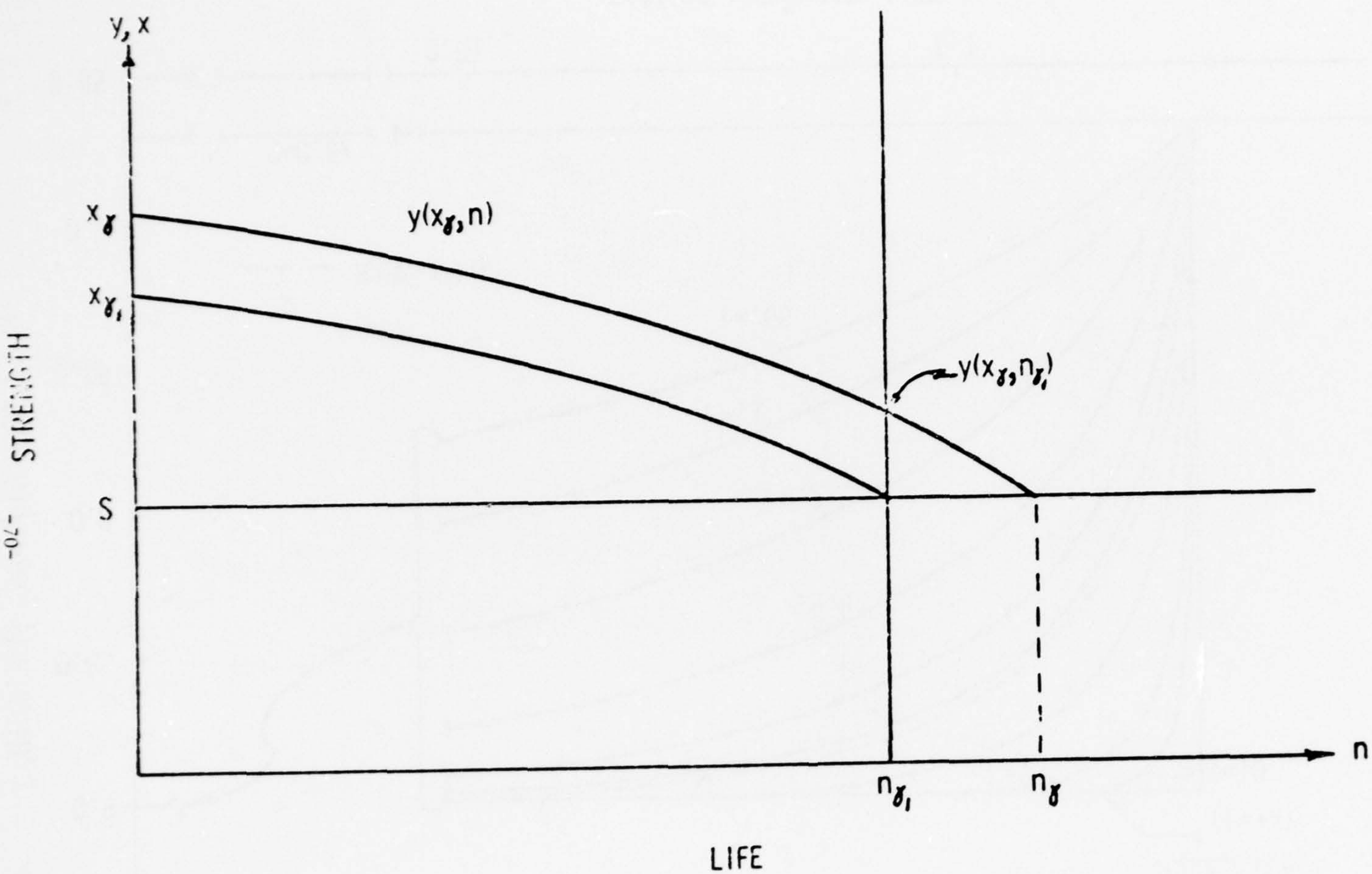

Figure 3. Schematic of Specimen Residual Strength vs. Life.

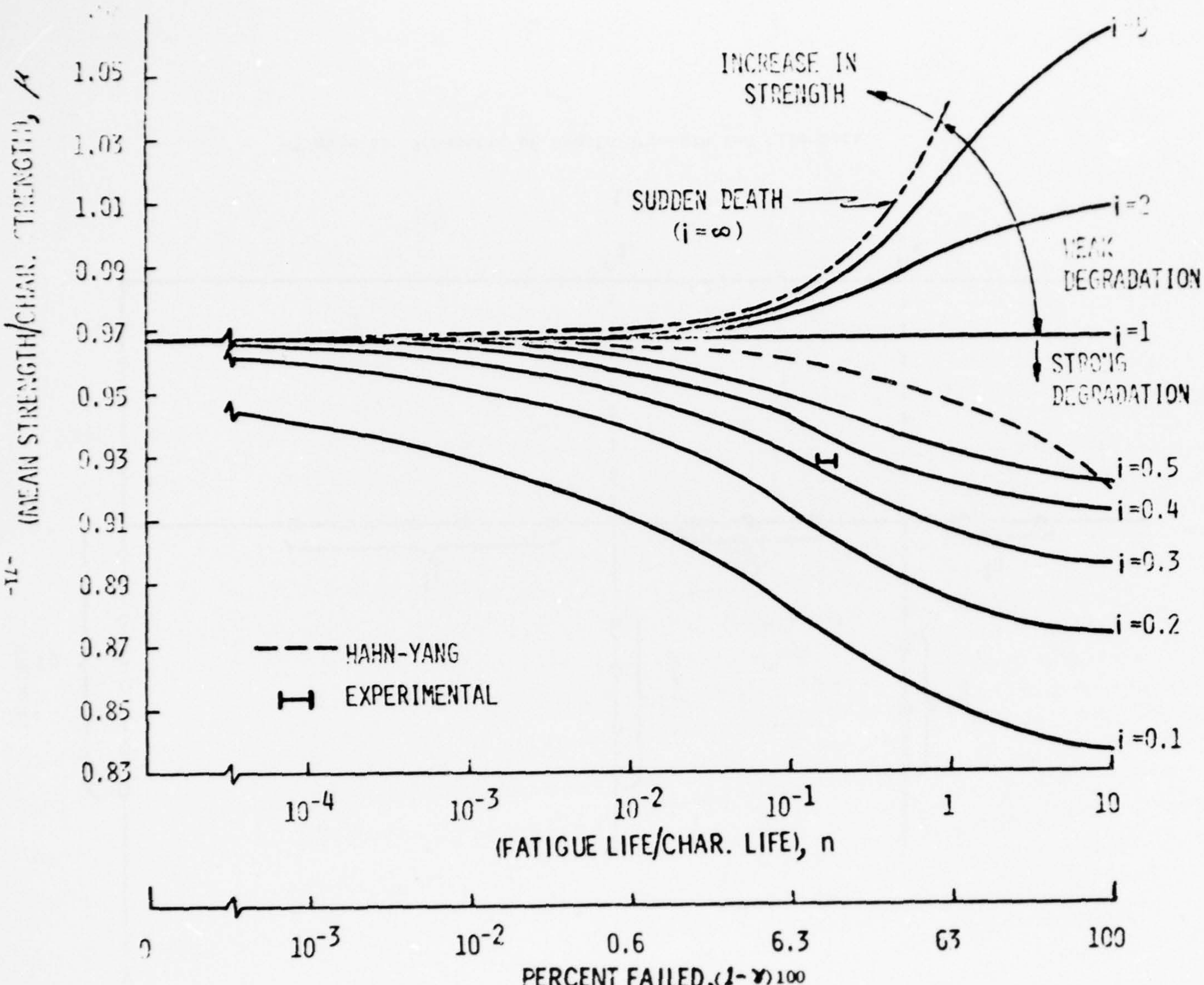

Figure 4. Mean Residual Strength vs. Fatigue Life for the Present Approach with $\alpha = 15.3$, $\alpha_1 = 1.18$, and $S = 0.676$. Data from Yang-Liu [3].

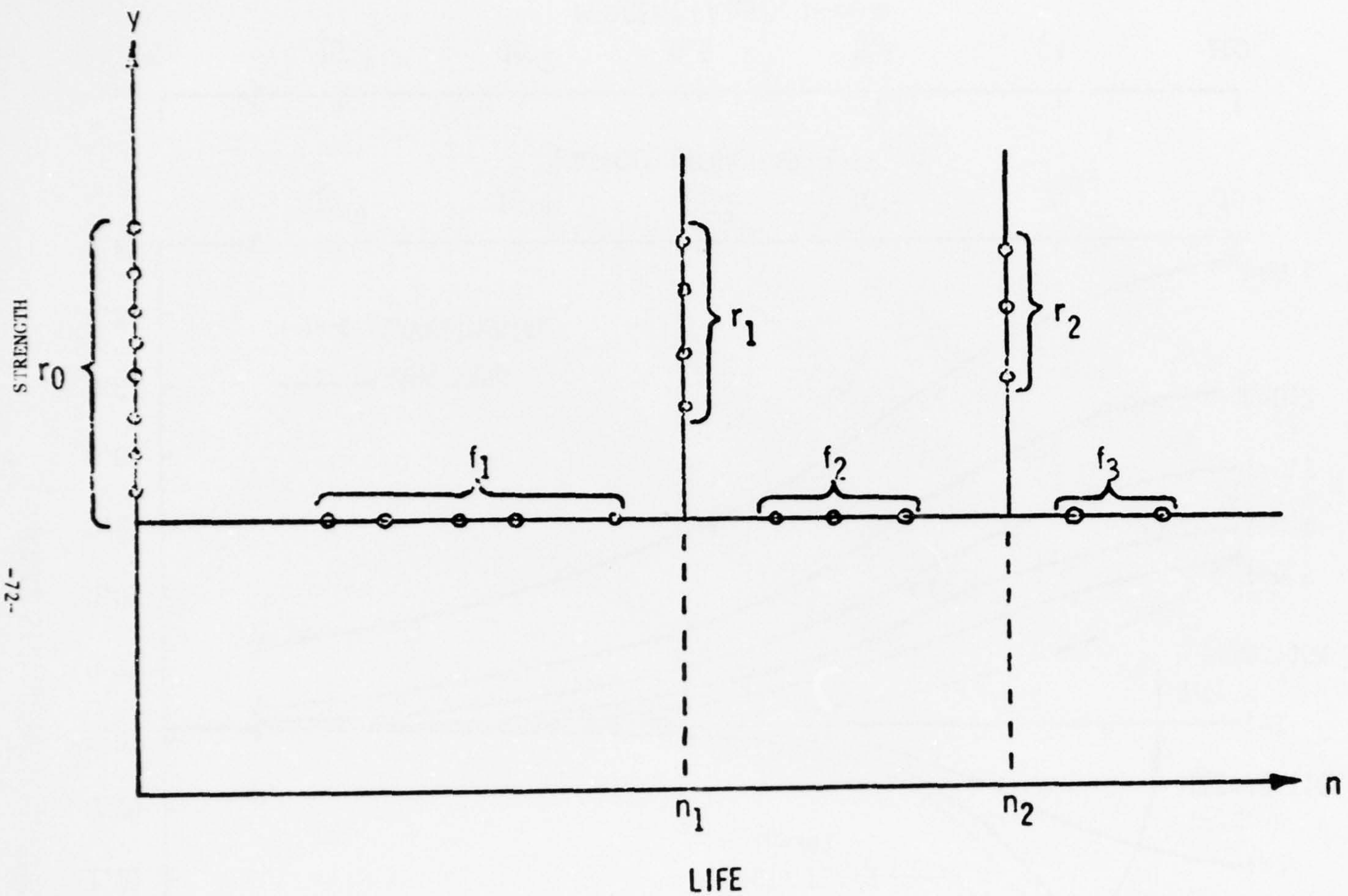

Figure 5. Schematic of Sample Strength and Life Data.

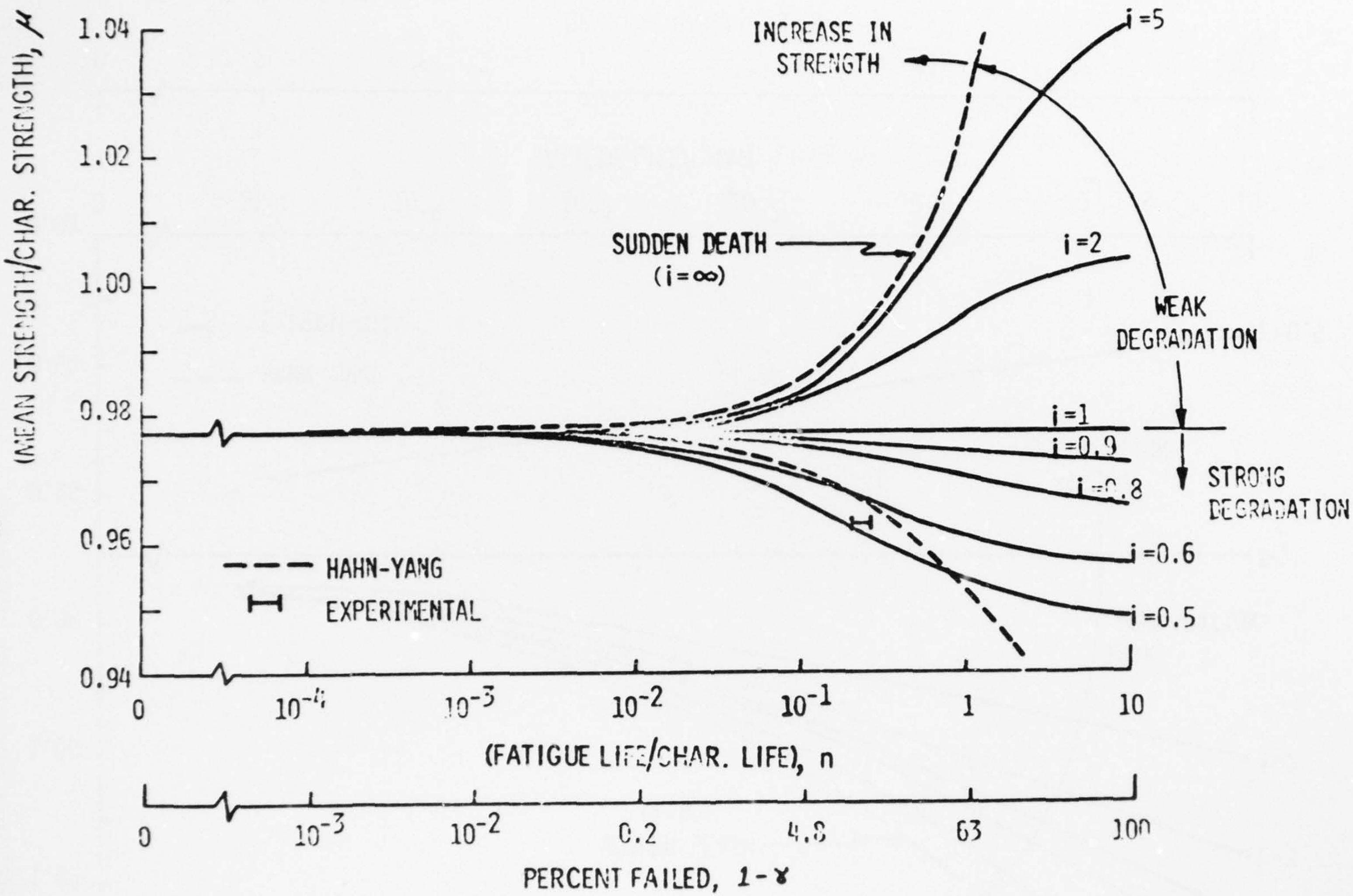

Figure 6. Mean Residual Strength vs. Fatigue Life for the Present Approach with $\alpha = 23.9$, $\alpha_1 = 1.31$, and $S = 0.7$. Data from Ryder-Walker Laminate I [5].

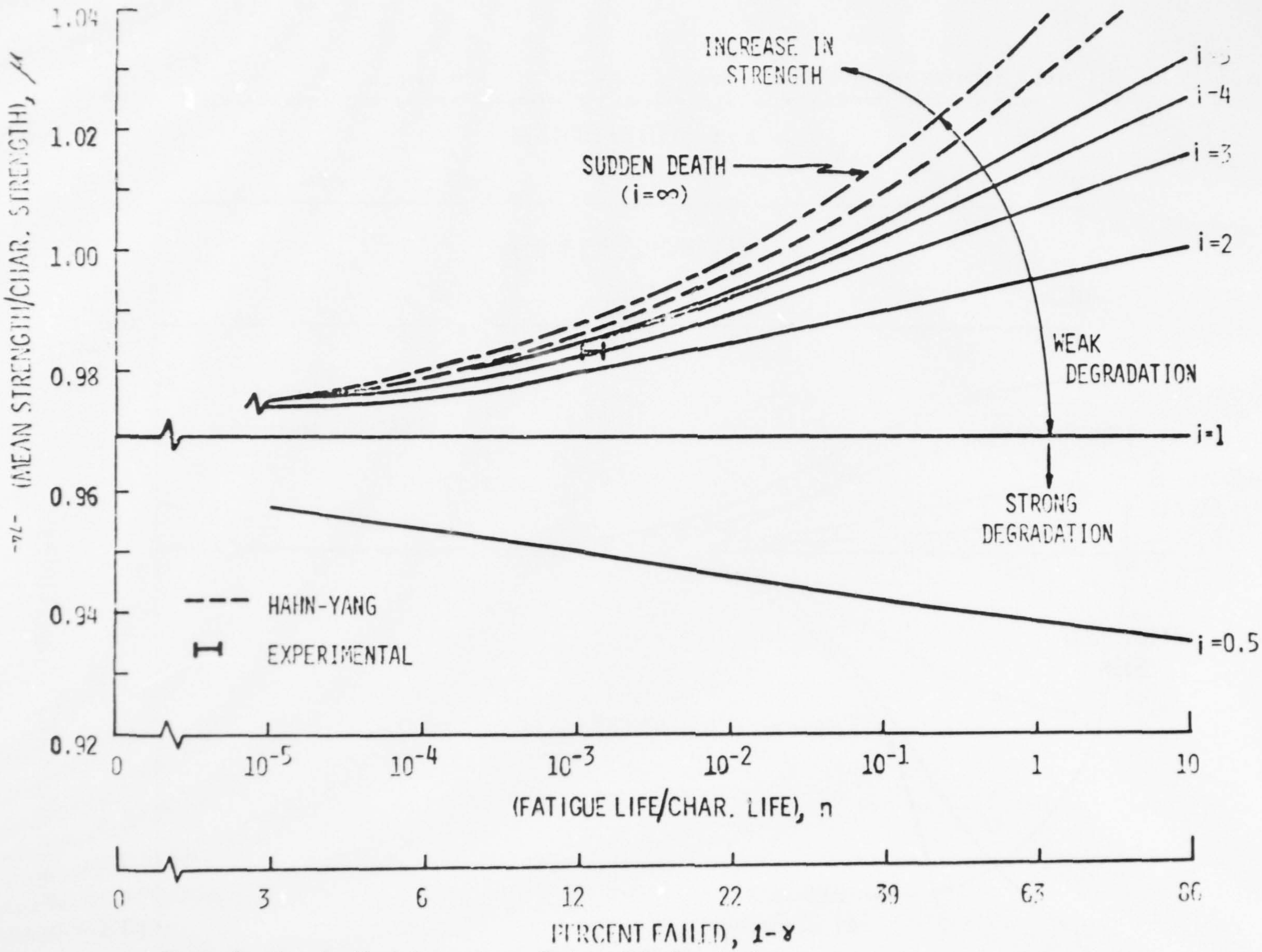

Figure 7. Mean Residual Strength vs. Fatigue Life for the Present Approach with α = 16.8, α_1 = 0.30, and S = 0.685. Data from Ryder-Walker Laminate II [5].

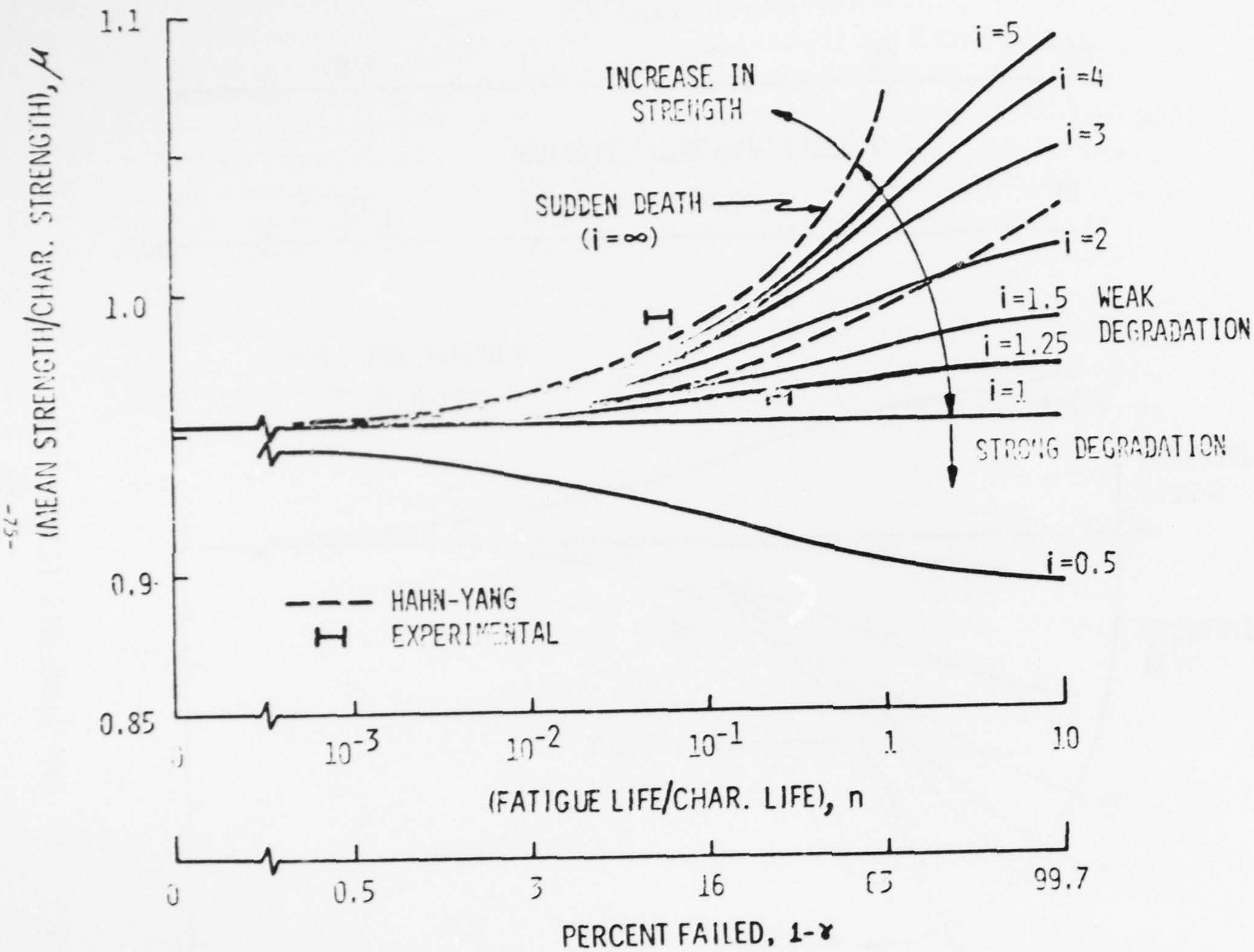

Figure 8. Mean Residual Strength vs. Fatigue Life for the Present Approach with $\alpha = 9.7$, $\alpha_1 = 0.75$, and $S = 0.61$. Data from Awerbuch-Hahn [6].

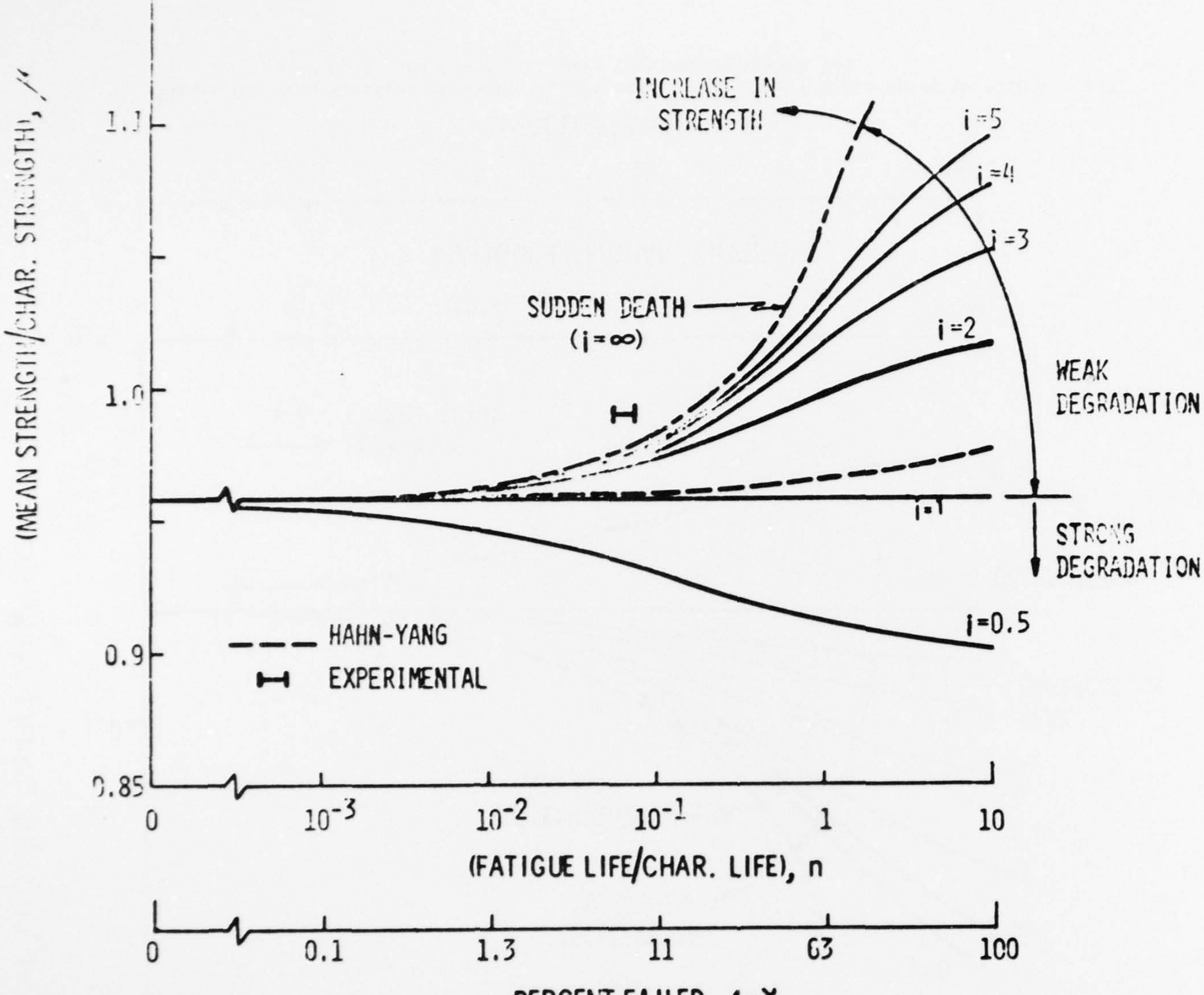

Figure 9. Mean Residual Strength vs. Fatigue Life for the Present Approach with $\alpha = 10.2$, $\alpha_1 = 0.94$, and $S = 0.67$. Data from Wang [7].

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